Infectious Disease Detection with Private Information

Alexander E. Saak

Markets, Trade and Institutions Division
INTERNATIONAL FOOD POLICY RESEARCH INSTITUTE

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AUTHOR

Alexander E. Saak, International Food Policy Research Institute
Research Fellow, Markets, Trade and Institutions Division
a.saak@cgiar.org
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ABSTRACT

In this paper we study incentives to report clinically suspect situations in a simple model of an infectious animal disease with limited diagnostic resource. We characterize a transfer scheme that sustains credible reporting and implements an efficient test allocation. In a game without monetary transfers, credible reporting and first-best targeted testing are achievable in both laissez-faire and efficient disease control regimes when the disease occurrence among few well-informed producers is unlikely. When the number of producers is small, random testing is optimal under mandatory depopulation of untested animals, but credible reporting can be necessary for testing to improve welfare under laissez-faire disease control if private information is sufficiently precise. When the number of producers is large, random testing always improves welfare, and if private information is precise, disease occurrence is unlikely, and testing capacity is small, efficient testing is achievable without transfers in the efficient disease control regime.

Keywords: infectious disease, reporting, diagnostic testing, indemnity design, private information
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1. INTRODUCTION

Early detection of an outbreak of an infectious disease—arguably the most important factor in reducing economic losses from a livestock disease epidemic—has received surprisingly little attention from economists (Hamilton and Bruckner 2010; FAO 2002). For many infectious animal diseases, livestock farmers are in the best position to report warning signs such as changes in animal behavior or increased mortality (Elbers et al. 2010; Catley, Alders, and Wood 2011). Yet, “governments have no control over whether a farmer decides to report a diseased or dead animal to a vet or government agricultural body” (Palmer, Fozdar, and Sully 2009, 361).

Although in many countries farmers are not directly compensated for reporting suspect cases, they typically receive indemnity payments for animals culled by the government (World Bank et al. 2006; Ott 2006; Institute of Medicine and National Research Council 2009). However, the effects of compensation that is primarily intended to enhance compliance with eradication measures on incentives to report are not always clear (Gramig, Horan, and Wolf 2009). For example, Kuchler and Hamm (2000) find evidence of delayed reporting in their empirical investigation of the response of the supply of infected animals reported by farmers to indemnity levels.2

In this paper we study incentives to report clinically suspect situations from an informational perspective. In our model, reports are cheap talk (Farrell and Rabin 1996), that is, costless and unverifiable messages that privately informed producers send to the social planner who allocates fixed diagnostic resources such as laboratory tests (active disease surveillance). To rely on producers' reports, the social planner offers a compensation scheme that incentivizes truth telling (passive disease surveillance). We take a mechanism design approach and look for an equilibrium that implements the first-best allocation of diagnostic tests. Our main result is that in a game without monetary transfers, credible communication and first-best targeted testing are achievable in both laissez-faire and efficient disease control regimes when the disease occurrence among few well-informed producers is unlikely. We also characterize an optimal compensation scheme when these conditions do not hold, and we consider the effects of testing in equilibrium with uninformative reports.

The basic idea explored in this paper is that in the absence of monetary transfers and government intervention in disease control beyond testing, the producer’s choice of whether to report or not report a suspect situation is the choice between higher and lower probability of being selected for a thorough clinical inspection. Whether producers gain more from better information about their own or their neighbors’ animals depends on the initial information about health and epidemiological parameters. On the one hand, the producer whose animals have been tested makes an informed disease control decision. On the other hand, the tested producer is exposed to a greater transmission risk from the remaining untested neighbors. The untested producers exercise less care because they take into account that the tested producers are less likely to spread the disease.

We first analyze the model in Section 2 with two producers. In Section 3, we consider equilibrium with laissez-faire disease control. We show that reports are credible without monetary transfers when truth telling increases (decreases) the likelihood of being selected for testing for producers for whom the expected gain from learning the disease status of their own animals is greater (respectively, smaller) than the expected loss from disease transmission from undiagnosed neighbors. However, when private information is noisy or disease occurrence is likely, truth telling is incentivized by transfer payments contingent on the report and (possibly) test result in order to achieve efficient learning about disease prevalence through targeted testing.

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1 A subdiscipline of veterinary epidemiology known as participatory epidemiology is concerned with the elicitation of information from livestock and veterinary practitioners, animal owners, and farmers to improve disease control programs (Catley, Alders, and Wood 2011). Here we study how economic considerations influence communication quality.

2 Also, Elbers et al. (1999, 2006) and Cuenot et al. (2003) compare the quantities of officially notified and expected clinically suspect situations and document instances of underreporting.
In Section 4, we consider a regime in which the health authority enforces efficient disease control policy. Then farmers’ reports become costly signals as their reports directly affect the cost of compliance with the policy. The enforcement of efficient disease control also increases the benefits of being selected for testing because the compliance costs are greater for the untested producers and because complying farms are less likely to infect farms that tested negative. Whether this makes it easier or more difficult to sustain credible reporting depends on whether the truth-telling incentive compatibility for producers with suspicions or without suspicions are binding in equilibrium with laissez-faire disease control. We find that private information is irrelevant and random testing is optimal when tested healthy animals are protected by the mandatory depopulation of untested animals.

In Section 5, we consider equilibrium with uninformative reports and demonstrate that although random testing improves welfare under efficient disease control, there are parameter values such that random testing decreases welfare under laissez-faire disease control when the number of producers is small and private information is sufficiently precise.

In Section 6, we investigate the model with a large number of producers. In this case, monetary transfers are always needed to sustain credible communication and efficient targeted testing under laissez-faire disease control. However, when private information is sufficiently precise, the disease occurrence is unlikely, and the diagnostic capacity is small, credible reporting and the first-best test allocation are achievable without transfers under mandatory efficient disease control. Even though an individual report has a negligible effect on the disease transmission risk, compliance with the mandatory disease control policy is costlier for farmers who report a suspect situation and receive a testing priority. We also show that random tests necessarily increase welfare whenever there are sufficiently many producers. In Section 7, we point out that in equilibrium the informativeness of the reports can vary across asymmetric producers.

To study the informational value of reporting, we consider a very simple model of an infectious animal disease with just two periods. In the first period animals on each farm exogenously become infected or remain healthy. Neither farmers nor the health authority know whether animals are infected, but farmers privately observe noisy signals about the health of their animals and decide whether to report them truthfully to the authority. The health authority has the ability to determine whether some but not all animals carry the disease before it spreads. In the second period, the health authority discloses its findings to farmers, and each farmer decides how many animals to keep or complies with the mandatory disease control program. Then the virus spreads from farms with sick animals to farms with initially healthy animals. Finally, farmers sell healthy animals, if any, and receive monetary transfers from the health authority (compensation for participating in disease surveillance).

Outbreaks of highly pathogenic avian influenza (HPAI) in the poultry industry, especially in developing countries, illustrate the key features of our modeling of (1) disease control, (2) epidemiology, (3) diagnostics, and (4) reporting. First, the life cycle of broilers (young chickens) is short and farmers can decrease disease risk by depopulating their flocks and temporarily shifting away from poultry production and toward other economic activities (Ifft, Roland-Host, and Zilberman 2010). Second, consistent with our modeling of disease epidemiology, HPAI is highly contagious and kills the entire flock when it becomes infected (Aral et al. 2010). Third, disease surveillance capacity including human resources, infrastructure, and laboratory facilities is limited in many developing countries (Azhar et al. 2010; Chan et al. 2010). Fourth, incentivizing disease reporting using compensation schemes is of practical concern. For example, World Bank in its guidelines for control of HPAI takes the position that enhancing “early reporting and complete culling of diseased or suspected birds is … the first objective of compensation schemes” (World Bank et al. 2006, ix).

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3 Our setting shares features with the two-period model of an infectious disease that Mesnard and Seabright (2009) use to study the effects of restricting migration by potentially infected individuals who have private information about their health.

4 In addition to depopulation, the risk of HPAI at the farm level can be affected by adopting biosecurity measures such as poultry housing, vaccination, and improved worker and poultry hygiene. We consider a small economy and ignore the equilibrium and direct effects of an HPAI outbreak on demand and the price of poultry as well as human health (Ifft, Roland-Host, and Zilberman 2010; Malani, Boni, and Galvani 2008).
Related Literature

To our knowledge, this is the first paper to study incentive compatibility of truthful reporting and optimal allocation of the testing resource in a model of an infectious disease with imperfect private information. Most of the literature on the economics of infectious animal diseases is concerned with actions that reduce disease risks and treats the information environment as exogenous to the model (Horan et al. 2010). This paper, however, endogenizes the benefits and costs of reporting and characterizes the smallest (positive) monetary transfers that make reporting credible and implement the efficient allocation.

The design of indemnity payments that incentivize producers to invest in biosecurity and report infection to the government in the presence of asymmetric information has recently been discussed in Gramig, Horan, and Wolf (2009). However, unlike in the present paper, in their model the benefits and costs of reporting are exogenous, there is no epidemiological dynamics, and the farmer knows the true disease prevalence in the herd before reporting. Also, Sheriff and Osgood (2010) consider the effects of cash transfers and testing on the sellers’ incentives to disclose exogenous food safety that is correlated over time but not across sellers.

This paper is most closely related to Malani and Laxminarayan (2011), who investigate whether a country that privately observes a positive test result from an imperfect diagnostic test for an outbreak discloses that result to its trading partner. In their model the epidemic spreads in one direction, and additional public information about whether there was an outbreak is exogenous and revealed before the reporting decision is made. Our model differs from theirs in that we consider multiple report senders (farmers); an epidemic that can spread in any direction; and endogenous diagnostic testing that follows, rather than precedes, reporting.

In both models, in principle, it is always possible to design a compensation scheme that incentivizes truthful reporting since the government (or trading partner) observes both the reports and the test results (or consequences of an outbreak). However, not all compensation schemes may be implementable. In the context of reporting at the country level studied in Malani and Laxminarayan (2011), imposing greater sanctions on the country that suffers an epidemic if it reported a negative rather than a positive test result is not sequentially optimal. In the context of reporting at the producer level, compensation schemes may be constrained by the budgetary considerations and administrative costs as well as limited producer’s financial assets that rule out imposing large penalties (World Bank et al. 2006).

More generally, our analysis is related to a model of cheap talk with multiple senders with partial, non-overlapping, and complementary private information in McGee and Yang (2009). Although we consider a very simple signal space, we allow for two-way communication. In our model cheap talk is followed by strategic learning and disclosure by the receiver (the health authority that improves information received from senders) and decisionmaking (disease control) by the senders (producers).
2. MODEL

The model used in this paper has \( N \geq 2 \) identical risk-neutral producers (senders), indexed by \( i \in \{1, \ldots, N\} \), and a diagnostian (health authority). A producer can be one farmer or a larger collective decisionmaking unit such as a village. There are two time periods, \( t = 0,1 \). In period 0, each producer owns one unit of stock of either healthy (H) or sick (S) animals, where \( S \equiv 0 \) and \( H \equiv 1 \). In period 1, each producer \( i \) keeps \( n_i \in [0,1] \) animals at constant cost \( c > 0 \) per animal and disposes of the rest at no cost.\(^5\) We will also refer to \( n_i \) as the disease control effort.\(^6\)

Let \( \widetilde{\theta}_{i,t} \in \{S, H\} \) denote the random state of health of producer \( i \)'s animals in period \( t \) with a typical realization \( \theta_{i,t} \).\(^7\) Producers cannot distinguish between healthy and sick animals before choosing \( n_i \). In the end of period 1, producer \( i \) privately learns her animals' final health state \( \theta_{i,1} \); sick animals die and have zero value, and healthy animals are sold at price \( p > c \) per unit. Her payoff is given by

\[
p \theta_{i,1} n_i - cn_i, \quad i = 1, \ldots, N. \tag{1}
\]

**Disease Epidemiology**

In period 0 producer \( i \)'s animals are healthy with probability \( \alpha \) or sick with the complementary probability \( 1 - \alpha \), that is,

\[
\Pr(\widetilde{\theta}_{i,0} = S) = 1 - \alpha, \quad i = 1, \ldots, N, \tag{2}
\]

where \( \widetilde{\theta}_{1,0}, \ldots, \widetilde{\theta}_{N,0} \) are drawn independently. In period 1 the probability that producer \( i \)'s animals are sick is given by

\[
\Pr(\widetilde{\theta}_{i,1} = S | \widetilde{\theta}_{i,0} = \theta_{i,0}, \ldots, \widetilde{\theta}_{N,0} = \theta_{N,0}) = \theta_{i,0} \frac{\delta}{N - 1} \sum_{j=1}^{N} n_j (1 - \theta_{j,0}) + 1 - \theta_{i,0}, \quad i = 1, \ldots, N, \tag{3}
\]

for any \( \theta_{1,0}, \ldots, \theta_{N,0} \), where \( \delta \in [0,1] \) is the infectiousness of the disease. In accordance with (3), animals that are sick in period 0 remain sick in period 1. In addition, in period 1 the disease can spread between producers. Consistent with the traditional models of infectious diseases (for example, Horan et al. 2010), we assume that the probability that initially healthy animals become sick is proportional to the number of initially sick animals.\(^8\)

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\(^5\) That is, producers are able to shift out of animal production into other activities with the zero expected return. The results do not change qualitatively when the "cost" of disposal is positive or negative but sufficiently small.

\(^6\) We consider disease control only through depopulation, but with minor changes, our model admits an alternative interpretation in which \( 1 - n_i \) is the number of vaccinated (rather than removed) animals at cost of \( p - c \) per animal and \( n_j \) is the number of susceptible (and possibly infectious) animals.

\(^7\) That is, the disease spreads quickly within a farm (or a village) so that in each period all of a producer’s (or village’s) animals are either healthy or sick.

\(^8\) Our results continue to hold when a producer’s animals can become sick in period 1 even when none are sick in period 0.
Private Information in Period 0

In period 0 each producer $i$ privately observes a random signal correlated with her animals’ initial health, $\theta_{i,0}$, denoted by $Y_i \in \{s, h\}$ with a typical realization $y_i$. The producers’ signals are independently and identically distributed and are imperfectly informative. Signals are correct with probability $\beta \in (\frac{1}{2}, 1)$ and incorrect with probability $1 - \beta$, that is, $\Pr(Y_i = h | \theta_{i,0} = H) = \Pr(Y_i = s | \theta_{i,0} = S) = \beta$ and $\Pr(Y_i = s | \theta_{i,0} = H) = \Pr(Y_i = h | \theta_{i,0} = S) = 1 - \beta$ for each $i$. Signal $h$ (respectively, signal $s$) indicates that the producer’s animals are less likely to be sick (respectively, more likely) than according to the prior. For example, signal $s$ (suspicions) is observed when a farmer notices symptoms or unusual behavior of her animals or knows about their prior exposure to the disease, and signal $h$ is observed when a farmer does not have reason to be suspicious. It will be convenient to let $\lambda_s = \Pr(\theta_{i,0} = H | Y_i = y)$ denote the producer’s posterior belief, where $\lambda_s = \frac{\alpha(1-\beta)}{\alpha(1-\beta) + (1-\alpha)\beta}$ and $\lambda_h = \frac{\alpha\beta}{\alpha\beta + (1-\alpha)(1-\beta)}$. Also, let $g_s = \Pr(Y_i = s) = \alpha(1-\beta) + (1-\alpha)\beta$ and $g_h = \Pr(Y_i = h) = \alpha\beta + (1-\alpha)(1-\beta)$ denote the probabilities with which producers observe signals $s$ and $h$.

Reports, Testing, Diagnostician’s Disclosure, and Transfers

In period 0, the diagnostician can at no cost determine the initial state of health $\theta_{i,0}$ of any producer’s herd (using a diagnostic test with perfect sensitivity and specificity) but can test at most $M$ herds, where $1 \leq M < N$. Upon observing $y_i$, each producer sends to the diagnostician a message (report) $r_i \in \{s, h\}$.

Then the diagnostician chooses a subset of producers $T$ for diagnostic testing, where either $T = \emptyset$, which means that none of the producers are tested, or $T \subset \{1, \ldots, N\}$ and $|T| \leq M$. Let $T(r_1, \ldots, r_N) : \{s, h\}^N \to \{T, T^c\} \subset \{1, \ldots, N\}$, $|T| \leq M \cup \emptyset$ denote the diagnostician’s testing strategy.

In period 1, the diagnostician sends a (possibly producer-specific) message $m_i \in \{r_1, \ldots, r_N, T, \theta_{j,0}\}_{j \in T}$ to each producer $i \in \{1, \ldots, N\}$ and a transfer $\tau_j \geq 0$ to each tested producer $j \in T$. The messages may include other producers’ reports, the identities of the tested producers, and the test results. Transfers received by the tested producers are chosen in accordance with the predetermined function $\tau : \{s, h\} \times \{S, H\} \to [0, \infty)$, that is, $\tau_j = \tau(r_j, \theta_{j,0})$ for each $j \in T$. These monetary transfers do not directly affect welfare, that is, the sum of producers’ payoffs net of transfers, but they can influence the producers’ reporting decisions. All of this is common knowledge. Each producer $i$ uses the message from the diagnostician, $m_i$, her private signal, $y_{i,0}$, and her report, $r_i$, to update her beliefs about the initial states of health of all producers’ animals.

The timing of events is shown in Figure 2.1.

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9 The analysis does not change as long as tests are sufficiently precise.

10 Report $r_i = h$ can be interpreted as the decision not to report, and $r_i = s$ as the decision to report one’s suspicions.
We will consider two regimes: with laissez-faire depopulation and with mandatory depopulation. Under laissez-faire depopulation, producers choose \( n_i \)'s simultaneously and noncooperatively. Under mandatory depopulation, the producers must choose the socially efficient herd sizes conditional on the information available to the diagnostician.\(^{11}\)

We first consider the implementation of the first-best test allocation under laissez-faire depopulation. As usual, the game is solved by backward induction. We begin by determining herd sizes in non-cooperative equilibrium after testing in period 1. Then we find testing and disclosure policies, \( T \) and \( m_1, ..., m_N \), in period 0 that maximize expected welfare 

\[
W(\theta_1, ..., \theta_N, T) = E[\sum_{i=1}^{N} p_{i} \Theta_{i} - cn_{i} | y_1, ..., y_N]
\]

assuming that producers reveal their signals \( y_1, ..., y_N \) in a subgame perfect equilibrium. The final step is to characterize a transfer policy \( \tau \) with the smallest possible expected payments that implements a socially efficient allocation of diagnostic tests in a perfect Bayesian equilibrium.

Next we will analyze the case with \( N = 2 \) and \( M = 1 \).

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\(^{11}\) The regime with government intervention corresponds to a disease surveillance and control program wherein the government not only tests animals for infectious diseases but also monitors or administers biosecurity measures such as preemptive culling.
3. TWO PRODUCERS, $N = 2$

Laissez-Faire Depopulation following Testing

Suppose that in period 1 it is commonly known that producer $T = i$ has been tested and learned the test result, $\theta_{i,0}$. Then, as will be shown next, equilibrium in period 1 is unique and producers achieve the same payoffs for any beliefs about one another’s private information. This property will simplify our characterization of the optimal diagnostician’s testing and disclosure policies.

Let $\sigma_y \in [0,1]$ denote producer $i$’s belief that untested producer $j \neq i$ observed signal $y$, $y \in \{s, h\}$. A producer who tested positive ($\theta_{i,0} = S$) achieves the highest payoff if she removes all animals, $n_i = 0$, for any belief $\sigma_y$. The reason is that a producer with sick animals in period 0 that remain sick in period 1 earns $-cn_i < 0$ if she keeps $n_i > 0$ animals but earns zero (the reservation payoff) by depopulating her herd. This implies that the untested producer knows that there is no disease transmission risk; that is, her profits from keeping $n_j$ animals are just $p\theta_{j,0}n_j - cn_j$, because a neighbor who tested positive removes all her animals and a neighbor who tested negative is not infectious. Hence, the untested producer chooses $n_j$ to maximize $pE[\tilde{\theta}_{j,0} | Y_j = y_j]n_j - cn_j$, or

$$\max_{n_i \in [0,1]} p\lambda_y n_j - cn_j,$$

and the (privately) optimal depopulation strategy for the untested producer is given by $n_j = n^u(y_j)$, where

$$n^u(y) = \begin{cases} 0, & \text{if } \lambda_y < \frac{c}{p} \\ \tilde{n}_y \in [0,1], & \text{if } \lambda_y = \frac{c}{p}, \ y \in \{s, h\} \\ 1, & \text{if } \lambda_y > \frac{c}{p} \end{cases}.$$

(4)

To simplify the presentation we will assume that an untested producer with signal $y$ removes all animals, $\tilde{n}_y = 0$, if $\lambda_y = \frac{c}{p}$, $y \in \{s, h\}$. Note that the laissez-faire depopulation strategy of the untested producer in (4) is independent of the test result.

Finally, we determine the optimal herd size for a producer who tested negative. Using (3) and (4), her expected payoff is given by

$$\max_{n_i} p(1 - \delta \sum_{y \in \{s, h\}} n^u(y)(1 - \lambda_y)\sigma_y)n_i - cn_i.$$

Here, $\delta \sum_{y \in \{s, h\}} n^u(y)(1 - \lambda_y)\sigma_y$ is the probability that the initially healthy animals of producer $i$ will become infected by the untested animals of producer $j$ when the tested producer believes that the untested producer observed signal $y$ with probability $\sigma_y$. Differentiation with respect to herd size $n_i$ yields

$$p(1 - \delta \sum_{y \in \{s, h\}} n^u(y)(1 - \lambda_y)\sigma_y) - c$$

$$\geq \min_{\sigma \in [0,1]} (p(1 - \delta n^u(s)(1 - \lambda_s)) - c)\sigma + (p(1 - \delta n^u(h)(1 - \lambda_h)) - c)(1 - \sigma) \geq 0,$$
where the last inequality follows by (4). Therefore, keeping all animals, \( n_i = 1 \), is an optimal choice for a producer who tested negative for any belief function \( \sigma_i \). Let \( n^d(\theta) = \theta \) denote the equilibrium depopulation strategy of a tested producer whose animals’ initial state of health is \( \theta \in \{S, H\} \). Therefore, we have Lemma 1, as follows:

**Lemma 1** (Belief-free equilibrium under laissez-faire depopulation in period 1): Suppose that \( N = 2 \); one of the producers is tested, \( T \in \{1, 2\} \); and it is commonly known that the tested producer learns the test result, \( \theta_{T,0} \). Then equilibrium herd sizes are unique and independent of the producers’ beliefs about one another’s private information.

It is worthwhile to point out that the two convenient properties—(a) the number of animals kept by the tested producer is independent of her beliefs about the private information of the untested producer, and (b) the number of animals kept by the untested producer is independent of the test result—hold because we assumed that the marginal costs of keeping and disposing of animals are constant and the test is perfect.

To examine the testing policy, we will need to find the expected welfare before the test result \( \theta_{T,0} \) is known but after producers observed their signals. By Lemma 1, expected welfare after the test is given by

\[
\max[\lambda_{y_j} p - c, 0]
\]

when the tested producer has sick animals, which happens with probability \( 1 - \lambda_{y_j} \), and

\[
\begin{cases}
2(\lambda_{y_j} p - c) + (1 - \lambda_{y_j})(1 - \delta) p, & \text{if } \lambda_{y_j} > \frac{c}{p} \\
p - c, & \text{if } \lambda_{y_j} \leq \frac{c}{p}
\end{cases}
\]

when the tested producer’s animals are healthy, which happens with probability \( \lambda_{y_j} \). Therefore, the expected welfare before producer 1 is tested in laissez-faire equilibrium (without government disease control intervention) is given by

\[
W(y_1, y_2, 1) = \begin{cases}
(1 + \lambda_{y_1})(\lambda_{y_1} p - c) + \lambda_{y_1} (1 - \lambda_{y_2})(1 - \delta) p, & \text{if } \lambda_{y_1} > \frac{c}{p} \\
\lambda_{y_1} (p - c), & \text{if } \lambda_{y_1} \leq \frac{c}{p}
\end{cases}
\]

(5)

Note that, by symmetry, expected welfare before producer 2 is tested can simply be found as

\[
W(y_2, y_1, 1) = W(y_2, y_1, 1).
\]

**Targeted Testing with Informative Reports**

We now look for a test allocation in period 0 that maximizes the overall utilitarian welfare assuming that the social planner observes the producers’ signals \((y_1, y_2)\). We start by finding an optimal test allocation assuming that it is optimal to test one of the producers, \( T^* (y_1, y_2) \in \{1, 2\} \). Then will we show that, in fact, higher welfare is achieved under prioritized testing than when none of the producers are tested, \( \max[W(y_1, y_2, 1), W(y_1, y_2, 2)] \geq W(y_1, y_2, \emptyset) \).
**To Test Producer 1 or Producer 2?**

We start by considering the determinants of the individual incremental payoff from being selected for testing. On the one hand, diagnosed producer $i$ gains the option value generated by information about her own herd’s health keeping the disease transmission risk constant (at zero since, by Lemma 1, the untested producer is not exposed to the transmission risk for $N = 2$):

$$V(y_i) \equiv E[(p\hat{\theta}_{i,0} - c)n^d(\hat{\theta}_{i,0})|Y_i = y_i] - E[(p\tilde{\theta}_{i,0} - c)n^d(\tilde{\theta}_{i,0})|Y_i = y_i]$$

$$= \lambda_{y_i} (p - c) - \max[p\lambda_{y_i} - c, 0] = \min[(1 - \lambda_{y_i})c, \lambda_{y_i} (p - c)] > 0.$$  

The gain from an informed depopulation decision, $V(y_i)$, is greatest when the production cost, $c$, is not too high or too low. The option value arises because a diagnosed producer does not depopulate an initially healthy herd when $\frac{c}{p} \geq \lambda_{y_i}$ or keep sick animals when $\frac{c}{p} < \lambda_{y_i}$. However, the tested producer $i$ also becomes exposed to the risk of disease transmission from her untested neighbor $j$:

$$C(y_i, y_j) = E[(p\tilde{\theta}_{i,0} - c)n^d(\tilde{\theta}_{i,0})|Y_i = y_i]$$

$$- E[(p\tilde{\theta}_{i,0}(1 - \delta n^u(\tilde{y}_j)(1 - \tilde{\theta}_{j,0}))) - c)n^d(\tilde{\theta}_{i,0})|Y_i = y_i, Y_j = y_j]$$

$$= p\delta E[n^d(\tilde{\theta}_{i,0})n^u(\tilde{y}_j\tilde{\theta}_{i,0}(1 - \tilde{\theta}_{j,0})|y_{i}, y_{j}) = \begin{cases} \frac{p\delta\lambda_{y_j}(1 - \lambda_{y_i})}{p} > 0, & \text{if } \lambda_{y_j} > \frac{c}{p} \\ 0, & \text{if } \lambda_{y_j} \leq \frac{c}{p} \end{cases}.$$  

There is no increase in the transmission risk when the untested producer removes all animals. Otherwise, an untested producer can infect an initially healthy tested producer. The expected loss from disease transmission conditional on $y_i, y_j$ is the price of a healthy animal, $p$, multiplied by the probability that the tested producer is healthy, $\lambda_{y_i}$, times the probability that the untested animals are sick and infectious, $\delta n^u(\tilde{y}_j|1 - \lambda_{y_j})$. Similarly, the untested producer $j$ foregoes the gain from becoming informed about the initial state of health of her own animals, $-V(y_j)$, but is also shielded from the transmission risk, $C(y_j, y_i)$, since a tested neighbor removes sick animals.

The *incremental social* welfare from testing producer 1 rather than producer 2 can now be written as

$$W(y_1, y_2, 1) - W(y_1, y_2, 2) = V(y_1) - V(y_2) - C(y_1, y_2) + C(y_2, y_1).$$  

The socially efficient choice of whether to test a producer with signal $y_1$ or $y_2$ is determined by comparing the expected gains from informed depopulation decisions, $V(y_1)$ and $V(y_2)$, and the expected losses due to disease transmission risk, $C(y_1, y_2)$ and $C(y_2, y_1)$.

Let $\varphi(y_1, y_2) : \{s, h\} \times \{s, h\} \rightarrow [0, 1]$ denote the probability with which producer 1 is tested, $\Pr(T(y_1, y_2) = 1) = \varphi$ and $\Pr(T(y_1, y_2) = 2) = 1 - \varphi$. When producers observe the same signals, $(y_1, y_2) \in \{(s, s), (h, h)\}$, it is trivially inconsequential which producer is tested, and random testing is optimal, $\varphi^*(s, s) = \varphi^*(h, h) = \frac{1}{2}$. So suppose that producers observed different signals, $(y_1, y_2) \in \{(s, h), (h, s)\}$. It will be convenient to refer to a producer with signal $h$ ($s$) as an $h$-producer ($s$-producer). Let us first compare the option values of diagnosis for an $h$-producer and an $s$-producer: $V(h) \geq (s)V(s)$ as

$$\frac{c}{p} \geq (s)\frac{\lambda_{y_j}}{1 - \lambda_{y_j} + \lambda_{y_i}}.$$  

9
If the relative production cost is sufficiently high, an \( h \)-producer gains more from knowing the initial state of health of her animals because she is more likely to increase her output upon learning it. However, if the relative production cost is sufficiently low, then an \( s \)-producer gains more from being diagnosed because she is more likely to save on the production costs. Also, let us compare the transmission risks that a tested \( h \)- and \( s \)-producer are exposed to:

\[
C(h, s) > C(s, h) > 0 \quad \text{if} \quad \lambda_s > \frac{p}{c},
\]
\[
C(h, s) = 0 < C(s, h) \quad \text{if} \quad \lambda_s \leq \frac{p}{c} < \lambda_h, \quad \text{and}
\]
\[
C(h, s) = C(s, h) = 0 \quad \text{if} \quad \lambda_h \leq \frac{p}{c}.
\]

The transmission risk is greater when an \( h \)-producer is tested if an untested \( s \)-producer keeps her animals. However, it is greater when an \( s \)-producer is tested if an untested \( s \)-producer depopulates her herd while an untested \( h \)-producer keeps hers. Finally, neither type of tested producer is exposed to the transmission risk when both types of untested producers depopulate their herds.

Now we are in a position to determine the socially efficient testing priority. Suppose that \( \frac{p}{c} < \max[k, \lambda_s] \), where \( k = \lambda_s \frac{(1-\lambda_h)(1-x_s^c) + \lambda_h}{1-\lambda_h + \lambda_s} \). Then we have \( W(s, h, 1) > W(s, h, 2) \), and it is socially efficient to test the \( s \)-producer, \( \varphi^*(s, h) = 1 - \varphi^*(h, s) = 1. \) When \( \lambda_s < \frac{p}{c} \leq k \), the incremental option value from informing the \( s \)-producer rather than the \( h \)-producer, \( V(s) - V(h) = \lambda_s (p-c) - (1-\lambda_h) c > 0 \), exceeds the incremental transmission risk, \( C(s, h) - C(h, s) = p\delta \lambda_s (1-\lambda_s) > 0 \). When \( \frac{p}{c} \leq \lambda_s \), not only does the \( s \)-producer gain more from a diagnosis, \( V(s) - V(h) = (1-\lambda_s) c - (1-\lambda_h) c > 0 \), but she is also exposed to a smaller transmission risk, \( C(s, h) - C(h, s) = p\delta \lambda_s (1-\lambda_h) - p\delta \lambda_h (1-\lambda_s) < 0 \).

However, for \( \frac{p}{c} > \max[k, \lambda_s] \), we have \( W(s, h, 1) < W(s, h, 2) \), and the \( h \)-producer receives the testing priority, \( \varphi^*(s, h) = 1 - \varphi^*(h, s) = 0. \) When \( \max[\lambda_s, k] < \frac{p}{c} < \lambda_h \), the incremental option value from informing the \( h \)-producer rather than the \( s \)-producer, \( V(h) - V(s) = (1-\lambda_h) c - \lambda_s (p-c) \geq (\leq) 0 \), is complemented (offset) by the reduction in the transmission risk, \( C(h, s) - C(s, h) = -p\delta \lambda_s (1-\lambda_h) < 0 \). When \( \frac{p}{c} \geq \lambda_h \), any untested producer removes her animals and therefore does not impose a transmission risk on the tested producer, \( C(h, s) = C(s, h) = 0. \) However, the option value of learning is greater for the \( h \)-producer compared with the \( s \)-producer, \( V(h) - V(s) = \lambda_h (p-c) - \lambda_s (p-c) = (\lambda_h - \lambda_s) (p-c) \geq (\leq) 0 \), because the \( h \)-producer’s animals are more likely to be initially healthy. This demonstrates that it is not necessarily socially efficient to allocate a limited diagnostic capacity to a producer whose animals are more likely to carry the disease. In our model, optimal risk-based testing targets “low-risk areas” when untested producers voluntarily depopulate their herds.

**To Test or Not to Test?**

We now verify that targeted testing \( \varphi^*(y_1, y_2) \) improves welfare (recall that, at most, one of the producers can be tested, \( M = 1 \)). That is, we establish that the sum of payoffs in equilibrium in which it is commonly known that one of the producers knows whether her animals carry disease in period 0 are always greater than the maximum payoffs achievable in equilibrium in which it is commonly known that none of the producers are tested. Furthermore, we show that this is true for any producers’ beliefs about one another’s private information. The proof of Lemma 2 is in Appendix A.
Lemma 2 (Optimal targeted testing versus no testing): Suppose that producers reveal their private signals. Then in equilibrium with laissez-faire depopulation, testing and disclosure policies exist such that the expected welfare is greater when one of the producers is tested than when none are tested, 
\[ \max[W(y_1, y_2, 1), W(y_1, y_2, 2)] > W(y_1, y_2, \emptyset) \] for all \( y_1, y_2 \).

Lemma 2 establishes that better information about local disease incidence is socially valuable when testing is prioritized based on the private information. If none of the producers are tested but private information is revealed, the social planner needs to decide whether to disclose information collected by passive surveillance (producers’ signals). However, in the proof of Lemma 2 we show that for each pair of the private signals, the expected social welfare under laissez-faire depopulation and optimal targeted testing is greater than that achieved under socially efficient depopulation but without testing. Since the latter is an upper bound on welfare in equilibrium without testing, this confirms that the optimal targeted testing, \( \Pr(T^* = 1) = \varphi^*(y_1, y_2) \) and \( \Pr(T^* = 2) = 1 - \varphi^*(y_1, y_2) \), dominates the no testing policy, \( T = \emptyset \). Note that, as we will show in Section 5, the revelation of private information is necessary for this result.\(^{12}\)

Diagnostician’s Disclosure

Here we show that an optimal diagnostician’s disclosure policy \( (m^*_1, m^*_2) \) is any disclosure such that \( \{T^*, \theta_{T^*,0} \} \in m^*_T \) for all \( \theta_{T^*,0} \in \{S, H\} \) and \( T^* \in \{1,2\} \). First, note that withholding the test result from the tested producer is not sequentially optimal because the informed tested producer removes sick animals (a socially optimal depopulation strategy). Second, the diagnostician cannot persuade the untested producer to keep fewer animals in equilibrium since, by assumption, the diagnostician’s messages are verifiable. By Lemma 1, the equilibrium laissez-faire depopulation strategy of the untested producer is independent of the test result and of the beliefs of the tested producer about the private information of the untested producer. Therefore, the only way for the diagnostician to manipulate his message to the untested producer is by withholding \( T, T \notin m_j, j \neq T \). However, by Lemma 2, in a perfect Bayesian equilibrium with the revelation of private information, each producer \( i \)'s (possibly out-of-equilibrium) belief is that producer \( j \neq i \) has been tested whenever \( \{T\} \notin m^*_i \). In that case, following the rules of Bayesian updating, an untested player infers that the other player has been tested if the identity of the tested player \( T \) is not disclosed to her. This demonstrates that in equilibrium the untested producer’s depopulation decision is the same for any (verifiable) message from the diagnostician.

Therefore, the greatest achievable welfare in equilibrium with the revelation of \( (y_1, y_2) \) is given by \( W^*(y_1, y_2) = \max[W(y_1, y_2, 1), W(y_1, y_2, 2)] \). It is achieved if the diagnostician follows testing policy \( \varphi^*(y_1, y_2) \) and disclosure policy \( (m^*_1, m^*_2) \) that notifies the tested producer of the test result.

Lemma 3 (Optimal testing and disclosure): The highest achievable level of welfare in any subgame perfect Bayesian equilibrium with laissez-faire depopulation is given by \( W^*(y_1, y_2) \) for all \( y_1, y_2 \). It is achieved if the reports are truthful and the diagnostician tests in accordance with \( \varphi^* \) and sends messages in accordance with \( (m^*_1, m^*_2) \).

---

\(^{12}\) In Section 5 we analyze the equilibrium in which reports are not credible and random testing or no testing are the only two feasible testing policies.
From Lemma 3 it follows that to achieve the first-best test allocation, the social planner needs to solve a mechanism design problem since the revelation of the producers’ signals is necessary for targeted testing. Because producers anticipate that the diagnostician will use their reports to allocate the diagnostic test, producers may want to deviate from truth telling.

Next we will examine whether the implementation of the first-best test allocation requires transfers that are contingent on the reports and (possibly) test result.

**Mechanism Design Problem**

To investigate the incentive compatibility of truthful reporting, we will need the following notation. By Lemma 1, in any sequentially optimal equilibrium, the expected (before the tested producer learns the test result but after the producers know which producer will be tested) payoffs (net of transfers) for a tested producer with signal $y'$ and an untested producer with signal $y$ are, respectively, given by

$$
\pi^d(y', y) = \begin{cases} 
\lambda_y'(p(1 - \delta(1 - \lambda_y)) - c), & \text{if } \lambda_y > \frac{p}{\delta} \\
\lambda_y'(p - c), & \text{if } \lambda_y \leq \frac{p}{\delta}
\end{cases}
$$

and

$$
\pi^n(y) = \begin{cases} 
p\lambda_y - c, & \text{if } \lambda_y > \frac{p}{\delta} \\
0, & \text{if } \lambda_y \leq \frac{p}{\delta}
\end{cases}
$$

Also, let $r(y): \{s, h\} \rightarrow \{s, h\}$ denote a pure reporting strategy of a producer with signal $y$. We look for an equilibrium in which reports are informative, $r(s) \neq r(h)$, and without loss of generality, we focus on an equilibrium in which reports are truthful, $r^*(y) = y$. Then the diagnostician’s belief that producer $i$’s animals are healthy is simply given by $\lambda_i (= \lambda_{y_i})$ as if the diagnostician directly observed the producers’ signals.

Upon observing $y_i$, producer $i$ does not update her beliefs about the initial state of health of her neighbor’s animals because $(\tilde{\theta}_0, \tilde{\theta}_0)$ and $(Y_1, Y_2)$ are drawn independently across producers. Therefore, if producer $i$ reports $r \in \{s, h\}$, her total earnings can be written as

$$
\pi(y_i, r) = \sum_{y' \in \{r, h\}} g_{y'} (\varphi^*(r, y') (\pi^d(y_i, y') + E[\tau(r, \tilde{\theta}_0, 0) | Y_i = y_i]) + (1 - \varphi^*(r, y')) \pi^n(y_i)),
$$

where $E[\tau(r, \tilde{\theta}_0, 0) | Y_i = y_i] = (1 - \lambda_{y_i}) \tau(r, S) + \lambda_{y_i} \tau(r, H)$ is the expected transfer from the diagnostician. To understand this expression, note that in an equilibrium with truthful reporting, producer $i$ is tested and earns $\pi^d(y_i, y') + E[\tau(r, \tilde{\theta}_0, 0) | Y_i = y_i]$ with probability $\varphi^*(r, y')$ and is not tested and earns $\pi^n(y_i)$ with probability $1 - \varphi^*(r, y')$ when the other producer reports $y'$, which occurs with probability $g_{y'}$.

Producers choose whether to report $h$ or $s$ to manipulate the test allocation, and they report truthfully, $r^*(y) = y$ for all $y \in \{s, h\}$, if and only if

$$
\pi(y, y) \geq \pi(y, y') \quad \text{for all } y, y' \in \{s, h\}.
$$

We look for a transfer policy $\tau^*$ such that (6a) holds with the minimum expected transfers.

---

13 That is, except for a zero probability event that condition $\frac{p}{\delta} = k \geq \lambda_y$ holds and random testing is optimal.
subject to (6a).\textsuperscript{14}

**Producer’s Reporting Decision**

To further understand the producers’ incentives to report, consider the incremental payoff net of transfers from reporting a signal that increases the likelihood of being selected for testing. Suppose that under testing policy $\phi^*$, the diagnostician tests a producer who reported $y^d$ if producers report $y^d$ and $y^u$; that is, $W(y^d, y^u, 1) > W(y^d, y^u, 2)$. Then by reporting $y^d$ instead of $y^u$, producer $i$ increases her expected payoff net of transfers by

$$
\Delta(y_i) = \sum_{y_i \in \{S,H\}} (V(y_i) - C(y_i, y'))(\phi^*(y^d, y') - \phi^*(y^u, y'))g_{y'}
= (V(y_i) - C(y_i, y^d))\frac{1}{2}g_{y'} + (V(y_i) - C(y_i, y^u))\frac{1}{2}g_{y'} = \frac{1}{2}(V(y_i) - E[C(y_i, Y_i)]),
$$

where $E[C(y_i, Y_j)] = \delta_p E[u^d(\tilde{\theta}_{i,0})\tilde{\theta}_{i,0}^n(y_j)(1 - \tilde{\theta}_{j,0}) | Y_i = y_i], j \neq i$. The probability of selection increases by just $\frac{1}{2}$ because producers are symmetric and equally likely to be selected for testing when they report the same signals.

In addition, a tested producer receives a (possibly zero) transfer contingent on her report and test result. The incremental expected transfer from reporting $y^d$ rather than $y^u$ is given by

$$
P(y_i) = \frac{1}{2}g_{y'} + g_{y'} E[\tau^*(y^d, \tilde{\theta}_{i,0}) | Y_i = y_i] - \frac{1}{2}g_{y'} E[\tau^*(y^u, \tilde{\theta}_{i,0}) | Y_i = y_i],
$$

where $\frac{1}{2}g_{y'} + g_{y'}$ is the probability of being selected for testing after reporting $y^d$ and $\frac{1}{2}g_{y'}$ is the probability of being selected for testing after reporting $y^u$. So the total incremental payoff from reporting $y^d$ instead of $y^u$ can be written as $\pi(y_i, y^d) - \pi(y_i, y^u) = \Delta(y_i) + P(y_i)$.

Let us refer to transfers that are independent of the test result, that is, transfers such that $\tau(s, S) = \tau(s, H) \text{ and } \tau(h, S) = \tau(h, H)$, as simple transfers. From the linear structure of the problem (6) it immediately follows that an optimal transfer scheme has the following properties: When $\Delta(y^u) \leq 0 \leq \Delta(y^d)$, truth telling is incentive compatible without transfers; $\tau^*(y, \theta) = 0$ for all $\theta$, $y$, and $P(y^u) = P(y^d) = 0$. When $\Delta(y^u) \leq \Delta(y^d) < 0$ (or $0 < \Delta(y^u) \leq \Delta(y^d)$), a simple transfer scheme $\tau^*(y^d, S) = \tau^*(y^d, H) > 0 = \tau^*(y^u, S) = \tau^*(y^u, H)$ (or $\tau^*(y^u, S) = \tau^*(y^u, H) > 0 = \tau^*(y^d, S)$ $= \tau^*(y^d, H)$) that satisfies $\Delta(y^u) + P(y^u) \leq 0 = \Delta(y^d) + P(y^d)$ (or $\Delta(y^u) + P(y^u)$ $= 0 \leq \Delta(y^d) + P(y^d)$) can make truth telling incentive compatible. However, when $\Delta(y^u) > \Delta(y^d)$, transfers that induce truth telling must be contingent on the report and the test result: $\tau^*(s, S) \geq \tau^*(s, H) = 0$ and $\tau^*(h, S) \geq \tau^*(h, H) = 0$ with at least one strict inequality, and satisfy $\Delta(y^u) + P(y^u) \leq 0 \leq \Delta(y^d) + P(y^d)$ with at least one strict equality.

\textsuperscript{14}This criterion can be motivated by administrative costs that increase with the amount transferred (and possibly the complexity of the transfer scheme). For simplicity, we ignore such costs in our model. If they are added to the model, an additional consideration will be whether the benefits from testing that is based on truthful reports offset the costs of administering a compensation policy that makes truthful reporting incentive compatible.
We begin with a straightforward case when the disease is noninfectious. We then proceed to analyze the reporting decisions and optimal transfers when the disease is infectious.

**Noninfectious Disease, \( \delta = 0 \)**

For \( \delta = 0 \) both types of producers prefer to be selected for testing because \( \Delta(y) = \frac{1}{2} V(y) > 0, \ y = s, h \). When the probability of disease transmission is close to zero, producers’ main concern is to avoid keeping sick animals rather than to protect their animals from disease that can be transmitted from the other producer in period 1.

For \( \frac{k}{p} \geq k \), the option value of diagnosis is greater for an \( h \)-producer than for an \( s \)-producer, \( \Delta(h) = \frac{1}{2} \lambda_h (p - c) > \frac{1}{2} \lambda_s (p - c) = \Delta(s) > 0 \). Because \( h \)-producers receive the testing priority, only \( s \)-producers want to deviate from truthful reporting. However, because an \( s \)-producer gains less from reporting \( h \) compared with an \( h \)-producer, simple transfers can make truth telling incentive compatible. The smallest such transfers are given by

\[
\tau^*(s, S) = \tau^*(s, H) = \frac{\Delta(s)}{\frac{1}{2} g_s} > 0 = \tau^*(h, S) = \tau^*(h, H).
\]

(7)

Under this transfer scheme, the incentive compatibility constraint for an \( s \)-producer binds while an \( h \)-producer strictly prefers to report \( h \): \( \Delta(s) + P(s) = 0 < \Delta(h) + P(h) \), where \( P(s) = P(h) = -\Delta(s) \).

It is worth pointing out that in this case the following transfers also support truthful reporting:

\[
\tau(h, S) = \tau(h, H) = 0, \tau(s, S) = \frac{\Delta(s)}{\frac{1}{2} g_s (1 - \lambda_s)} (1 - \rho), \ \tau(s, H) = \frac{\Delta(s)}{\frac{1}{2} g_s \lambda_s} \rho \text{ for any } \rho \in [0,1],
\]

and generate the same expected transfers of \( 2g(s)\Delta(s) \) as under transfer scheme \( \tau^* \) in (7).

The case with \( \frac{k}{p} < k \) is analogous. Now, in accordance with the optimal testing policy, \( s \)-producers receive the testing priority and \( h \)-producers need to be incentivized to tell the truth. In this case, the smallest simple transfers that make truth telling incentive compatible are given by

\[
\tau^*(h, S) = \tau^*(h, H) = \frac{\Delta(h)}{\frac{1}{2} g(h)} > 0 = \tau^*(s, S) = \tau^*(s, H).
\]

(8)

When the disease is infectious, an optimal transfer policy rewards reporting signal \( s \) or \( h \) or both, depending on the relative production cost, \( \frac{c}{p} \), the accuracy of the private information, \( \beta \), and the prior probability that the initial state of health is \( h \), \( \alpha \). The three sections that follow explain the three possibilities: an untested \( h \)-producer and an untested \( s \)-producer remove their animals, an untested \( h \)-producer keeps her animals but an untested \( s \)-producer removes hers, and untested \( h \)- and \( s \)-producers keep their animals. For simplicity, we set \( \delta = 1 \).15

**Untested \( h \)-Producer and \( s \)-Producer Depopulate, \( n^u(s) = n^u(h) = 0 \)**

The combinations of parameters \( \alpha \) and \( \frac{c}{p} \) for which condition \( \frac{k}{p} \geq \lambda_h \) holds and an untested \( h \)-producer and an untested \( s \)-producer depopulate their herds is depicted as area I in Figure 3.1 for \( \beta = 0.8 \)

---

15 The results continue to hold for sufficiently large values of \( \delta \).
(moderately informative private signals). As in the case with \( \delta = 0 \), when an untested producer removes her animals, the tested producer faces no transmission risk (except now the absence of transmission risk is endogenous). Therefore, a producer has nothing to gain when another producer is tested, and both \( h \)-producer and \( s \)-producer prefer to be selected for testing. Because testing is more valuable for an \( h \)-producer, tested \( s \)-producers receive transfer \( \tau^* (s, \cdot) \) in (7).

**Figure 3.1—Combinations of \( \alpha \) and \( \frac{c}{p} \)**

![Graph showing combinations of \( \alpha \) and \( \frac{c}{p} \)](Image)

*Source: Author’s creation.*

**Only Untested \( s \)-Producer Depopulates, \( n^u(s) = 0, n^u(h) = 1 \)**

When \( \lambda_s \leq \frac{c}{p} < \lambda_h \) (areas II–VI in Figure 3.1), an untested \( s \)-producer depopulates her herd but an untested \( h \)-producer keeps her animals. In this case, for an \( s \)-producer the option value generated by diagnosis is greater than the expected loss from disease transmission:

\[
\Delta(s) = \frac{1}{2} (\lambda_s (p-c) - (1-\alpha)(1-\beta)\lambda_s p) > 0 .
\]

(9)

For an \( h \)-producer, however, the incremental payoff from being selected for testing can be negative or positive:

\[
\Delta(h) = \frac{1}{2} (1-\lambda_h) c - (1-\alpha)(1-\beta)\lambda_h p \leq (>) 0 \text{ as } \frac{c}{p} \leq (>) \alpha \beta .
\]

(10)

To understand the second terms in (9) and (10), note that although a tested producer faces no transmission risk when her neighbor is an \( s \)-producer, an untested \( h \)-producer can be infectious. So the
expected loss due to transmission risk, \( E[C(y, Y_j)] = (1 - \alpha)(1 - \beta)\lambda, p \), is the product of (a) the probability that animals of an untested producer are sick but the producer observes signal \( h \), \( \Pr(\tilde{D}_{j0} = S, Y_j = h) = (1 - \alpha)(1 - \beta) \), (b) the probability that the animals of the tested producer are initially healthy, \( \lambda, \), and (c) the price of a healthy animal, \( p \).

Let us compare \( \Delta(s) \) and \( \Delta(h) \). When the production cost and the prior probability that animals are sick are both sufficiently small and private information is sufficiently precise, \( \Delta(h) \leq 0 \) and an \( h \)-producer prefers to remain uninformed but avoid being exposed to the transmission risk from an untested neighbor. However, when an \( h \)-producer prefers to be selected for testing, her incremental payoff may be greater or smaller than that for an \( s \)-producer:

\[
\Delta(h) \geq (\leq) \Delta(s) \quad \text{as} \quad \frac{z}{p} \geq (\leq) z \equiv \frac{\lambda_s + (1 - \alpha)(1 - \beta)(\lambda_h - \lambda_s)}{1 - \lambda_s + \lambda_s} \in (\lambda_s, \lambda_h). \quad (11)
\]

Although an \( h \)-producer values more the option value of diagnosis, \( V(s) = \lambda_s (p - c) < V(h) = (1 - \lambda_s) c \), she is also exposed to a greater transmission risk because she is more likely to have initially healthy animals, \( E[C(s, Y_j)] = (1 - \alpha)(1 - \beta) \lambda, p < (1 - \alpha)(1 - \beta)\lambda p = E[C(h, Y_j)] \).

Suppose that \( k < \frac{z}{p} < \lambda_h \) (areas II, III, and IV in Figure 3.1). Then \( h \)-producers receive the testing priority. Hence, from (9) it follows that \( s \)-producers must be compensated for revealing their type, because \( \Delta(s) > 0 \). From (11) it follows that whether simple transfers can make truth telling incentive compatible depends on \( \frac{z}{p} \geq (\leq) z \). For \( z \leq \frac{z}{p} \leq \lambda_h \) (area II in Figure 3.1) simple transfers in (8) are optimal because in this case, by (11), \( \Delta(h) \geq \Delta(s) > 0 \), and only the \( s \)-producer’s incentive compatibility constraint binds.

For \( k < \frac{z}{p} < z \) (areas III and IV in Figure 3.1), by (11), \( \Delta(s) > \Delta(h) \), which implies that transfers that make truth telling incentive compatible for both types must depend on the test result. A simple transfer triggered by an \( s \) report cannot sustain truthful reporting because it will necessarily violate either \( h \)-producer’s or \( s \)-producer’s incentive compatibility constraints. Therefore, we now look for transfers that are contingent on the test results.

For \( \max[\alpha, k] < \frac{z}{p} < z \) (area III in Figure 3.1), at optimum, only a tested \( s \)-producer receives a transfer, but the transfer must be greater when her animals are sick, \( \tau(s, S) > \tau(s, H) \). In particular, a transfer scheme that rewards only correct signal \( s \)

\[
\tau^*(s, S) = \frac{\Delta(s)}{\frac{z}{p} g_s(1 - \lambda_s)} > 0 = \tau^*(s, H) = \tau^*(h, S) = \tau^*(h, H)
\]

is optimal. For this range of parameters, although a test benefits more an \( s \)-producer, \( \Delta(s) > \Delta(h) \), welfare is greater when the \( h \)-producer is tested, \( W(h, s, 1) > W(h, s, 2) \).\(^\dagger\) Although an \( h \)-producer prefers to be selected for testing, \( \Delta(h) > 0 \), she is not swayed by compensation scheme \( \tau^* \) in (12), which pays out only to producers who report \( s \) and test positive. The expected diagnosis-contingent transfer from

\footnotetext[1]{Recall that in the truth-telling equilibrium, the social planner knows the private information of both producers but producers do not know one another’s signals when they evaluate gains from being selected for testing.}
reporting $s$ is greater for an $s$-producer than for an $h$-producer, and only $s$-producer’s incentive compatibility constraint binds, $\Delta(s) + P(s) = 0 < \Delta(h) + P(h)$, under transfer policy $\tau^*$ in (12).

For $\max[\lambda_s, k] < \frac{\alpha}{p} \leq \alpha$ (area IV in Figure 3.1), both truth-telling incentive compatibility constraints in (6a) bind at optimum, that is, $\Delta(s) + P(s) = \Delta(h) + P(h) = 0$, and the solution is unique with $\tau^*(s, H) = \tau^*(h, S) = 0$ and $\tau^*(s, S), \tau^*(h, H) > 0$, where $\tau^*(s, S)$ and $\tau^*(h, H)$ solve the following system of equations:

\[
\Delta(s) + P(s) = \Delta(s) + (g_s + \frac{1}{2} g_h)\lambda_s \tau^*(h, H) - \frac{1}{2} g_s (1 - \lambda_s) \tau^*(s, S) = 0, \tag{13a}
\]

\[
\Delta(h) + P(h) = \Delta(h) + (g_s + \frac{1}{2} g_h)\lambda_h \tau^*(h, H) - \frac{1}{2} g_s (1 - \lambda_h) \tau^*(s, S) = 0. \tag{13b}
\]

For $\max[\lambda_s, k] < \frac{\alpha}{p} \leq \alpha \beta$, both constraints bind because $\Delta(s) > 0 > \Delta(h)$ for this range of parameters. For $\alpha \beta < \frac{\alpha}{p} \leq \alpha$ we have $0 < \Delta(h) < \Delta(s)$, but $h$-producers place a sufficiently low value on being selected for testing and must be further incentivized to report $h$, given that reporting $s$ is rewarded.

When both $s$ and $h$ reports are rewarded by transfers, it must be that only the “correct” predictions trigger payments. To see why at optimum $\tau^*(s, H) = \tau^*(h, S) = 0$, consider a transfer scheme with $\tau(s, H) > 0$. Then increasing transfers for a correct prediction $\tau(s, S)$ by $\varepsilon / (1 - \lambda_s)$ for some small number $\varepsilon > 0$ and decreasing transfers for an incorrect prediction $\tau(s, H)$ by $\varepsilon / \lambda_s$ leaves the expected transfers and constraint $\Delta(s) + P(s) \leq 0$ unchanged. But this will make it easier to satisfy the constraint $\Delta(h) + P(h) \geq 0$ because an $h$-producer will receive $\tau(s, S) + \varepsilon / (1 - \lambda(s))$ with probability $1 - \lambda_h < 1 - \lambda$ and $\tau(s, H) - \varepsilon / \lambda_s$ with probability $\lambda_h > \lambda_s$ if she reports signal $s$. Therefore, at optimum, a transfer is received by the tested producer only when her report matches the test result. The transfers for correct predictions that solve the system of equations in (13) are given by

\[
\tau^*(s, S) = \frac{\lambda_h \Delta(s) - \lambda_s \Delta(h)}{\frac{1}{2} g_s (\lambda_h - \lambda_s)} , \quad \tau^*(h, H) = \frac{(1 - \lambda_s) \Delta(h) - (1 - \lambda_h) \Delta(s)}{(g_s + \frac{1}{2} g_h) (\lambda_h - \lambda_s)} .
\]

When $\lambda_s < \frac{\alpha \beta}{p} \leq k$ (areas V and VI in Figure 3.1), $s$-producers receive the testing priority. For $\max[\lambda_s, \alpha \beta] < \frac{\alpha}{p} \leq k$ (area V in Figure 3.1), an $h$-producer benefits from being tested herself, although less so than an $s$-producer, $\Delta(s) > \Delta(h) > 0$. The smallest simple transfer that incentivizes an $h$-producer to tell the truth is given by (8). These transfers do not violate the $s$-producer’s incentive compatibility constraint because, by (11), $\Delta(s) > \Delta(h)$ so that $\Delta(s) + P(s) > 0 = \Delta(h) + P(h)$ since the incremental expected transfer from reporting $s$ is $P(h) = P(s) < 0$ under transfer scheme $\tau^*$ in (8). In this case, diagnosis-contingent transfer schemes exist with $\tau(h, s), \tau(h, h) > 0 = \tau(s, s) = \tau(s, h)$, which also make truth telling incentive compatible and do not increase the expected transfers.

For $\lambda(s) < \frac{\alpha}{p} \leq \min[\alpha \beta, k]$ (area VI in Figure 3.1), no transfers are needed to make truth telling incentive compatible for both $h$-producer and $s$-producer because, by (9) and (10), $\Delta(h) \leq 0 < \Delta(s)$:

\[
\tau^*(s, S) = \tau^*(s, H) = \tau^*(h, S) = \tau^*(h, H) = 0. \tag{14}
\]
Now s-producers prefer to be selected for testing and h-producers prefer not to be selected for testing. This agrees with the preferences of the social planner.

**Untested Producers Do Not Depopulate, \( n^u(h) = n^u(s) = 1 \)**

When \( \frac{\bar{c}}{p} < \bar{\lambda} \) (areas VII and VIII in Figure 3.1), an untested s-producer and h-producer keep their animals. In this case, h-producers strictly prefer to remain untested

\[
\Delta(h) = \frac{1}{2}((1-\bar{\lambda}_h) c - (1-\alpha)\bar{\lambda}_h p) < 0
\]

because, by assumption, \( \frac{\bar{c}}{p} < \bar{\lambda}_h \). For s-producers the option value generated by diagnosis may be greater or smaller than the expected loss from transmission of disease from an untested producer:

\[
\Delta(s) = \frac{1}{2}((1-\bar{\lambda}_s) c - (1-\alpha)\bar{\lambda}_s p) \geq (>)0 \text{ as } \frac{\bar{c}}{p} \geq (>)\alpha \frac{1-\beta}{p}.
\]

To understand (15) and (16), note that because untested h- and s-producers keep their animals, the average probability that the untested animals are sick is \( 1-\alpha \). An s-producer gains more from being selected for testing, \( \Delta(s) > \Delta(h) \), because for an s-producer the value of the option generated by diagnosis is greater, \((1-\bar{\lambda}_s) c > (1-\bar{\lambda}_s) c\), and the expected loss from disease transmission is smaller, \((1-\alpha)\bar{\lambda}_s p < (1-\alpha)\bar{\lambda}_h p\).

In accordance with the optimal testing policy, s-producers receive the testing priority; so by (15), h-producer’s incentive compatibility constraint is easily satisfied, \( \Delta(h) < 0 \). By (16), \( \Delta(s) \geq 0 \), and reporting s is optimal for an s-producer for \( \alpha \frac{1-\beta}{p} \leq \frac{\bar{c}}{p} \leq \bar{\lambda}_s \) (area VII in Figure 3.1). For this range of parameters, producers can credibly communicate their signals without transfers (see 14).

However, for \( \frac{\bar{c}}{p} < \alpha \frac{1-\beta}{p} \) (area VIII in Figure 3.1), an s-producer needs to be incentivized by transfers since \( \Delta(s) < 0 \). The smallest simple transfers that make reporting credible are given by

\[
\tau^*(s, S) = \tau^*(s, H) = -\frac{\Delta(s)}{\frac{1}{2} g_s + g_h} > 0 = \tau^*(h, S) = \tau^*(h, H).
\]

A simple transfer policy is sufficient to ensure that both types of producers prefer to tell the truth because an s-producer loses less from being selected for testing, \( \Delta(h) < \Delta(s) < 0 \), so that at optimum, \( \Delta(h) + P(h) < 0 = \Delta(s) + P(s) \). The following transfer policies also support truthful reporting and generate the same expected transfers of \(-2g(s)\Delta(s)\) as transfer scheme \( \tau^* \) in (17):

\[
\tau(h, S) = \tau(h, H) = 0, \tau(s, S) = -\frac{\Delta(s)}{\frac{1}{2} g_s + g_h} \frac{1-\rho}{1-\bar{\lambda}_s}, \tau(s, H) = -\frac{\Delta(s)}{\frac{1}{2} g_s + g_h} \frac{\rho}{\bar{\lambda}_s}
\]

for all \( \rho \in [0,1] \). Summarizing the previous analysis we obtain Proposition 1.
**Proposition 1** (Implementation of first-best test allocation): Suppose that \( N = 2 \). Under transfer policy \( \tau^* \), a perfect Bayesian equilibrium exists with laissez-faire depopulation in which producers report truthfully, \( r^* \), and the diagnostician follows testing policy \( \varphi^* \) and disclosure policy \( (m_1^*, m_2^*) \), such that welfare \( W^*(y_1, y_2) \) is achieved for all \( y_1, y_2 \). Furthermore, the expected transfers are at least as great under any other transfer policy that allows producers to achieve the same level of welfare in a sequentially optimal equilibrium.

We established that report credibility is sustainable without transfers if and only if (areas VI and VII in Figure 3.1)

\[
\alpha \frac{1 - \beta}{p} \leq \frac{\xi}{p} \leq \max\{ \min[\alpha \beta, k], \lambda_1 \}. \tag{18}
\]

Condition (18) is easier to satisfy when the producers’ private information is more precise and the prior probability that animals are sick is smaller. For \( \beta \to 1 \) we have \( \lim_{\beta \to 1} k = \alpha^2 \), and (18) becomes \( 0 < \frac{\xi}{p} < \alpha^2 \). For \( \alpha \to 1 \) we have \( \lim_{\alpha \to 1} k = \beta \), and (18) becomes \( \frac{1 - \beta}{p} < \frac{\xi}{p} < 1 \). Therefore, we can state the following:

**Corollary** For sufficiently large \( \alpha \) and \( \beta \) there is a perfect Bayesian equilibrium with laissez-faire depopulation that implements the first-best allocation of the diagnostic test without transfers.

We find that a health authority does not need to offer rewards to rely on producers’ reports to target diagnostic efforts as long as the disease incursion happens infrequently and producers know well what the clinical signs of the disease look like. Also note that a necessary condition for (18) to hold is that \( s \)-producers receive the testing priority. For this range of parameters we can interpret the testing policy \( \varphi^* \) as prescribing a laboratory testing or a medical examination when farmers report clinically suspect cases and random sampling of animals for testing when there are no alarming reports from farmers.
4. EFFICIENT DEPOPULATION WITH INFORMATIVE REPORTS, \( N = 2 \)

Now we suppose that the social planner can enforce an efficient depopulation policy. As before, we start by determining the socially efficient herd sizes when the social planner observes the initial state of health of producer \( i \)'s animals, \( \theta_{i,0} \), and producer \( j \)'s signal \( y_j \). It is easy to check that the expected welfare conditional on that information is given by

\[
\theta_{i,0}(1 - \delta n_j (1 - \lambda_j) pn_i - cn_i + \lambda_j (1 - \delta n_i (1 - \theta_{i,0}))pn_j - cn_j ,
\]

and achieves its maximum at \( n_i = n^d(\theta_i) \) and \( n_j = n^{s*}(\theta_i, y_j) \), where

\[
n^{s*}(\theta, y) = \begin{cases} 0, & \text{if } \frac{p}{\delta} \geq \lambda_y - \theta \delta (1 - \lambda_y) \\ 1, & \text{if } \frac{p}{\delta} < \lambda_y - \theta \delta (1 - \lambda_y), \theta \in \{S, H\}, y \in \{s, h\}.
\end{cases}
\]

Here the superscript \( s \) stands for socially efficient.

Let us examine how laissez-faire depopulation strategies (in “Laissez-Faire Depopulation following Testing” in Section 3) deviate from the socially efficient strategies. The privately optimal choice of \( n^d(S) = 0 \) by a producer whose animals tested positive is also socially optimal because the private and social benefits of keeping sick animals are both zero while the private and social costs are strictly positive. The privately optimal choice of \( n^d(H) = 1 \) by a producer whose animals tested negative is also socially optimal because that producer does not impose a negative externality on the other producer. However, comparing \( n^{s*}(\theta_i, y_j) \) and \( n^s(y_j) \) reveals that the untested producer keeps too many animals relative to the socially efficient depopulation strategy if the tested producer's animals are healthy \( \theta_i = H \) and the production cost is not too high or too low, \( \lambda_{y_j} - \delta (1 - \lambda_{y_j}) \leq \frac{p}{\delta} < \lambda_{y_j} \).

Otherwise, the socially efficient and laissez-faire depopulation strategies of untested producers coincide as well.

Under efficient depopulation the maximum achievable expected welfare before testing is given by

\[
W^s(y_1, y_2, l) = \begin{cases} (1 + \lambda_{y_1})(\lambda_{y_2} p - c) + \lambda_{y_2} (1 - \lambda_{y_2})(1 - \delta)p, & \text{if } \frac{p}{\delta} < \lambda_{y_2} - \delta (1 - \lambda_{y_2}) \\ \lambda_{y_1} (p - c) + (1 - \lambda_{y_1})(\lambda_{y_2} p - c), & \text{if } \lambda_{y_2} - \delta (1 - \lambda_{y_2}) \leq \frac{p}{\delta} < \lambda_{y_2} \\ \lambda_{y_1} (p - c), & \text{if } \frac{p}{\delta} \geq \lambda_{y_2} \end{cases}
\]

if producer 1 is tested and \( W^s(y_1, y_2, 2) = W^s(y_2, y_1, l) \) if producer 2 is tested.

Let us remark that in our model incomplete information about the disease status is the sole source of negative externalities; that is, only untested producers keep too many animals, because the diagnostic test is perfect and the marginal costs of keeping and depopulating animals are constant. In fact, when either \( N = M \) (and all producers are tested and learn their test results) or private signals are perfect, \( \beta = 1 \), privately optimal depopulation decisions are also socially optimal.

Next we will consider an optimal allocation of the diagnostic test and incentives to report under mandatory efficient depopulation strategies conditional on the reports and test result when the testing capacity is limited, \( M = 1 \), and private information is imperfect, \( \beta < 1 \).
Testing and Reporting under Efficient Depopulation

Under efficient depopulation and private information revelation testing necessarily improves welfare because the social planner can always ignore the test result. As discussed in the previous section, for $\frac{\alpha}{p} \geq \lambda_h$ or $\frac{\alpha}{p} \leq \lambda_s - \delta(1 - \lambda_s)$ or $\frac{\alpha}{p} \leq \lambda_h - \delta(1 - \lambda_h)$, equilibrium testing, disclosure, and compensation schemes are the same as in equilibrium with laissez-faire depopulation simply because privately optimal depopulation strategies are also socially optimal.

For

$$\lambda_h - \delta(1 - \lambda_h) \leq \frac{\alpha}{p} < \lambda_s$$

(19)

both types of untested producers bear the cost of compliance with mandatory depopulation when the test result is negative. The untested producer would prefer to keep her animals but is forced to depopulate in accordance with the depopulation policy $n^{ua,s}(H, y) = 0$ (even though $n^{ua,s}(S, y) = 1$), $y \in \{s, h\}$. The social planner removes untested animals to protect the healthy animals because untested animals pose too great a disease transmission risk. In this case, the expected welfare before testing is the same whether an $s$-producer or an $h$-producer is tested:

$$W^*(s, h, 1) = \lambda_s (p - c) + (1 - \lambda_s)(\lambda_h p - c) = \lambda_h (p - c) + (1 - \lambda_h)(\lambda_s p - c) = W^*(s, h, 2).$$

The reason there is no difference whether the $s$-producer or $h$-producer is tested is that in either case one producer keeps her animals, and the overall probability that healthy animals are kept is exactly the same and equals $\lambda_s + (1 - \lambda_s)\lambda_h = \lambda_h + (1 - \lambda_h)\lambda_s = 1 - \lambda_h \lambda_s$. Therefore, when condition (19) holds, random testing is always efficient and the need for truthful reporting is obviated (and, trivially, no transfers are necessary to implement random testing). Note that (19) cannot hold unless private information is sufficiently noisy ($\beta$ is close to $\frac{1}{2}$) and the disease is sufficiently infectious ($\delta$ is close to 1).

Finally, suppose that $\lambda_s - \delta(1 - \lambda_s) < \frac{\alpha}{p} < \min[\lambda_s, \lambda_h - \delta(1 - \lambda_h)]$. Now only an untested $s$-producer incurs the compliance cost, $n^{ua,s}(H, s) = 0 < 1 = n^{ua,s}(H, h)$, and $s$-producers receive the testing priority since $W^*(s, h, 1) > W^*(s, h, 2)$. For $\delta = 1$ the incremental payoff from reporting $s$ rather than $h$ for a producer with signal $y$ is given by

$$\Delta(y) = \frac{\alpha}{2} \left( (1 - \lambda_s)c - (1 - \alpha)(1 - \beta)\lambda_y p - \alpha(1 - \beta)(\lambda_y p - c) \right) \geq (>) 0 \text{ as } \frac{\alpha}{p} \leq (>) \frac{(1 - \beta)y_i}{1 - \lambda_y + \alpha(1 - \beta)}. $$

The new term is the incremental cost of compliance with the mandatory depopulation policy. It consists of the payoff, $\lambda_y p - c$, that a producer who reported $s$ foregoes when her neighbor tests negative, which happens with probability $\frac{1}{2} \alpha(1 - \beta)$. The latter is the product of the probabilities of three conditionally independent events: the other producer is an $s$-producer ($g_s$), she is selected for testing ($\frac{1}{2}$), and her herd is healthy ($\lambda_y$). Note that an untested producer who reported $h$ is never subjected to mandatory depopulation.

Since $\Delta(h) < \Delta(s)$, a simple transfer scheme in (17) makes truthful reporting incentive compatible for $\frac{\alpha}{p} < \frac{(1 - \beta)y_i}{1 - \lambda_y + \alpha(1 - \beta)}$ (which implies that $\Delta(h) < \Delta(s) < 0$). Similarly, for $\frac{\alpha}{p} > \frac{(1 - \beta)y_i}{1 - \lambda_y + \alpha(1 - \beta)}$ (which implies that $0 < \Delta(h) < \Delta(s)$), a simple transfer scheme in (8) sustains credible reporting. However, for
\[ \frac{(1-\beta)\lambda_s}{1-\lambda_s + \alpha(1-\beta)} \leq \frac{c}{p} \leq \frac{(1-\beta)\lambda_h}{1-\lambda_h + \alpha(1-\beta)} \]  

(20)

reports are credible and the efficient allocation of the diagnostic test is implementable without transfers (see 14) under mandatory efficient depopulation, because (20) implies that \( \Delta(h) \leq 0 \leq \Delta(s) \).

Figure 4.1 shows that going from laissez-faire to efficient depopulation (when the two strategies differ) may make it easier or more difficult to satisfy the truth-telling incentive compatibility constraints. In equilibrium with laissez-faire depopulation reporting is credible without monetary transfers for the parameter values in areas II, III, IV, and V in Figure 4.1, where condition (18) holds, but credible reports require some transfers for the values of parameters outside of these areas. Let us now consider conditions such that reports are credible without monetary transfers under efficient depopulation. For the parameter values in areas I and II, condition (19) holds and monetary transfers are not necessary simply because it is efficient to test randomly and depopulate the herd of the untested producer whenever the animals of the tested producer are healthy. For parameter values in areas IV and VI, condition (20) holds and reporting is credible without monetary transfers under efficient targeted testing because the truth-telling incentive compatibility constraints for both types are slack.

**Figure 4.1—Credible reporting and depopulation compliance costs, \( \beta = 0.75, \delta = 1 \)**

Thus, for the parameter values in areas III and V, credible reporting is sustainable without monetary transfers under laissez-faire depopulation but not under efficient depopulation, while the opposite is true for areas I and VI.

**Proposition 2** (Efficient depopulation and credible reporting): Suppose that \( N = 2 \). Under efficient depopulation the first-best allocation can be achieved without monetary transfers if and only if condition (19) or condition (20) holds. These conditions are neither implied by nor imply condition (18), which is necessary and sufficient for sustainability of credible reporting under laissez-faire depopulation.
5. UNINFORMATIVE REPORTS, \( N = 2 \)

So far we focused on the first-best cheap talk equilibrium that sustains truth telling and implements the efficient test allocation. However, in the absence of monetary transfers, an equilibrium always exists in which producers’ reports are not credible. This happens if the social planner and producers expect the reported signals to be independent of what producers actually observe. Then reporting any signal independently of the observed signal is, in fact, a best response for each producer. Also, direct cheap talk between producers (where producers send reports to each other) cannot be both credible and influential because each producer prefers to claim to be the type that imposes a greater transmission risk to increase her neighbor’s incentives to remove animals.

Testing any producer necessarily increases welfare in equilibrium with uninformative reports and efficient depopulation because the social planner can always ignore the test result. However, as we will show next, under some conditions testing decreases welfare in equilibrium with uninformative reports and laissez-faire depopulation.

**Welfare under Laissez-Faire Depopulation, Uninformative Reports, and No Testing**

Let \( n^n(y) \) denote a best-response depopulation strategy for a \( y \)-producer in equilibrium without revelation of private information and testing. Here the superscript \( n \) stands for no revelation and no testing. Producer \( i \) now chooses \( n_i \) to maximize her expected payoff conditional only on her signal \( y_i \), so that

\[
n^n(y_i) = \arg \max_{n_i \in \{0,1\}} \sum_{y_j \in \{s,h\}} g_{y_j} ((\lambda_{y_j}(1-\delta)n^n(y_j))(1-\lambda_{y_j})p-c)n_i .
\]

It is easy to verify that the equilibrium depopulation strategy for an \( h \)-producer is given by

\[
n^n(h) = \begin{cases} 
0, & \text{if } \frac{\alpha}{p} \geq \lambda_h \\
\bar{n}_h, & \text{if } \lambda_h(1-\delta(1-\alpha)(1-\beta)) \leq \frac{\alpha}{p} < \lambda_h \\
1, & \text{if } \frac{\alpha}{p} < \lambda_h(1-\delta(1-\alpha)(1-\beta))
\end{cases}
\]

where \( \bar{n}_h \in (0,1] \) solves the indifference condition \( \lambda_h(1-\delta(1-\alpha)(1-\beta)\bar{n}_h) = \frac{\alpha}{p} \), and the equilibrium depopulation strategy for an \( s \)-producer is given by

\[
n^n(s) = \begin{cases} 
0, & \text{if } \frac{\alpha}{p} \geq \lambda_s(1-\delta(1-\alpha)(1-\beta)) \\
\bar{n}_s, & \text{if } \lambda_s(1-\delta(1-\alpha)) \leq \frac{\alpha}{p} < \lambda_s(1-\delta(1-\alpha)(1-\beta)) \\
1, & \text{if } \frac{\alpha}{p} < \lambda_s(1-\delta(1-\alpha))
\end{cases}
\]

where \( \bar{n}_s \in (0,1] \) solves the indifference condition \( \lambda_s(1-\delta(1-\alpha)(1-\beta + \beta \bar{n}_s)) = \frac{\alpha}{p} \). Also, let

\[
EW^n(Y_1, Y_2, \emptyset) = 2 \sum_{y_1 \in \{s,h\}} \sum_{y_2 \in \{s,h\}} g_{y_1}g_{y_2} ((\lambda_{y_1}(1-\delta)n^n(y_2))(1-\lambda_{y_2})p-c)n^n(y_1)
\]

denote the expected welfare (before producers observe their signals) in equilibrium without revelation of private signals and testing.

If \( \frac{\alpha}{p} \geq \lambda_h \), producers remove all animals (recall that now both producers are untested), and the expected welfare (before producers observe their signals) is \( EW^n(Y_1, Y_2, \emptyset) = 0 \). In this case, the disease risk is so high that even an \( h \)-producer depopulates her herd. If \( \lambda_h(1-\delta(1-\alpha)(1-\beta)) \leq \frac{\alpha}{p} < \lambda_h \), an \( s \)-
producer removes all animals, and an \( h \)-producer keeps \( \tilde{n}_h \in (0,1) \) animals (or randomizes between keeping all animals with probability \( \tilde{n}_h \) and removing all animals with probability \( 1 - \tilde{n}_h \)). In this case, the externality completely dissipates any expected social gains from production, \( EW''(Y_1, Y_2, \emptyset) = 0 \). If \( \lambda_s (1 - \delta (1 - \alpha)(1 - \beta)) \leq \frac{c}{p} < \lambda_h (1 - \delta (1 - \alpha)(1 - \beta)) \), then an \( s \)-producer removes all animals, an \( h \)-producer keeps all animals, and the expected welfare is

\[
E^*[W(Y_1, Y_2, \emptyset)] = g_s^2 2(\lambda_s (1 - \delta (1 - \lambda_s)) p - c) + 2g_s g_s (\lambda_h p - c) \\
= 2(\alpha \beta (1 - \delta (1 - \alpha)) (1 - \beta)) p - g_sc).
\] (21a)

If \( \lambda_s (1 - \delta (1 - \alpha)) < \frac{c}{p} < \lambda_s (1 - \delta (1 - \alpha)(1 - \beta)) \), then an \( h \)-producer keeps all animals, an \( s \)-producer keeps \( \tilde{n}_s \in (0,1) \) animals, and the expected welfare is

\[
E[W''(Y_1, Y_2, \emptyset)] = g_s^2 2(\lambda_s (1 - \delta (1 - \lambda_s)) p - c) + 2g_s g_s (\lambda_h (1 - \delta \tilde{n}_s (1 - \lambda_s)) p - c) \\
= 2c \frac{1 - \alpha}{1 - \beta} (2 \beta - 1).
\] (21b)

Finally, if \( \frac{c}{p} \leq \lambda_s (1 - \delta (1 - \alpha)) \), both types of producers keep their animals and the expected welfare is \( E[W''(Y_1, Y_2, \emptyset)] = 2(\alpha (1 - \delta (1 - \alpha)) p - c) \).

Under the partial resolution of uncertainty through testing, the negative production externality (excessive disease transmission risk) is asymmetric since only an untested producer keeps too many animals in equilibrium with laissez-faire depopulation. In contrast, when none of the producers are tested, producers take into account the transmission risk created by an (untested) neighbor and, on average, may keep fewer sick (as well as healthy) animals than a pair of tested and untested producers. As we will show next, a more symmetric externality that producers impose on each other under complete uncertainty about each other’s private information and actual initial states of health can alter the welfare economics of diagnostic testing.

**Welfare-Decreasing Random Testing**

When the social planner cannot rely on the reports to reveal producers’ signals and cannot enforce mandatory depopulation, by Lemma 1, the only policy choice is either (a) to randomly test and disclose the test result or (b) not to test. Note that under either testing policy there exists an equilibrium with uninformative reports. Under random testing, by (5), the expected welfare (before signals are observed) is given by

\[
E[\frac{1}{2} W(Y_1, Y_2, 1) + \frac{1}{2} W(Y_1, Y_2, 2)] = E[W(Y_1, Y_2, 1)] = \sum_{y_1, y_2 \in [R,H]} g_{y_1} g_{y_2} W(y_1, y_2, 1),
\]

because, by Lemma 1, when it is commonly known that one of the producers learns the actual initial state of health of her animals, the equilibrium outcome is independent of the producers’ beliefs about one another’s private information.

From our analysis of equilibrium without revelation and testing, it immediately follows that testing improves welfare, \( E[W(Y_1, Y_2, 1)] > E[W''(Y_1, Y_2, \emptyset)] \), for \( \frac{c}{p} \geq \lambda_h (1 - \delta (1 - \alpha)(1 - \beta)) \) or \( \frac{c}{p} < \lambda_s (1 - \delta (1 - \alpha)) \). If the former condition holds, either untested producers do not keep animals because the disease occurrence is too likely or the externality that producers impose on each other dissipates their expected payoffs, so that \( E[W''(Y_1, Y_2, \emptyset)] = 0 \) (reservation payoff). If the latter
condition holds, both types of producers keep their animals and impose the maximum possible externality on each other. Then improving information about the initial state of health for one of the producers cannot reduce welfare. Also, it is easy to verify that random testing improves welfare for any testing policy. Then the probability that an untested producer keeps sick animals is the same with or without testing, and testing necessarily improves welfare:

\[ W(y_1, y_2, 1) > W^*(y_1, y_2, \emptyset) \text{ for all } y_1, y_2, \text{ where} \]

\[
W(y_1, y_2, 1) = \begin{cases} 
\lambda_s (p - c), & \text{if } (y_1, y_2) = (s, s) \\
(1 + \lambda_h)(\lambda_h p - c), & \text{if } (y_1, y_2) = (s, h) \\
\lambda_h (p - c), & \text{if } (y_1, y_2) = (h, s) \\
(1 + \lambda_s)(\lambda_s p - c), & \text{if } (y_1, y_2) = (h, h) 
\end{cases}
\]

and

\[
W^*(y_1, y_2, \emptyset) = \begin{cases} 
0, & \text{if } (y_1, y_2) = (s, s) \\
\lambda_h p - c, & \text{if } (y_1, y_2) = (s, h) \\
\lambda_h p - c, & \text{if } (y_1, y_2) = (h, s) \\
2(\lambda_h^2 p - c), & \text{if } (y_1, y_2) = (h, h) 
\end{cases}
\]

So suppose that \( \lambda_s (1 - \delta (1 - \alpha)) \leq \frac{\alpha}{p} < \min[\lambda_h (1 - \delta (1 - \alpha)(1 - \beta)), \lambda_s] \). Then, by (5), the expected welfare under random testing is given by

\[ E[W(Y_1, Y_2, 1)] = (1 + \alpha)(ap - c) + \alpha(1 - \alpha)(1 - \delta)p . \] (22)

From (22) and our previous welfare calculations in (21), it follows that random testing decreases welfare,

\[ E[W^*(Y_1, Y_2, \emptyset)] \geq E[W(Y_1, Y_2, 1)] \], if and only if

\[
\max[\alpha \frac{2(1 - \beta - \delta(1 - \alpha)(1 - \delta(1 - \beta))}{3\alpha + \beta - 4\alpha \beta - 1}, \alpha(1 - \beta) \frac{2 - \delta(1 - \alpha)}{3\alpha + \beta - 5\alpha \beta - 1}] \leq \frac{\alpha}{p} < \lambda_s . \] (23)

Area I in Figure 5.1 illustrates the values of parameters \( \frac{\alpha}{p} \) and \( \beta \) such that condition (23) holds for \( \alpha = \frac{1}{2} \) and \( \delta = 1 \). In summary, we have established the following:

**Proposition 3** (Testing with uninformative reporting and two producers): Suppose that \( N = 2 \). In equilibrium with laissez-faire depopulation and uninformative reports, testing decreases welfare if and only if condition (23) holds.

A diagnosis has three effects on welfare in equilibrium with uninformative reports and laissez-faire depopulation. First, the tested producer benefits from an informed depopulation decision. Second, an untested producer benefits from the removal of animals that tested positive. Third, the tested producer is exposed to a greater transmission risk because the untested producer is more likely to keep sick animals in equilibrium with testing than in non-revealing equilibrium without testing. Suppose that an untested producer keeps her animals when her neighbor is tested but at least partially depopulates her herd in equilibrium without testing. Then, as illustrated in Figure 5, a range of parameter values exists such that the social cost from too many untested animals kept by an s-producer exceeds the social benefits of the test.
Note that random testing always improves welfare when producers’ private information is sufficiently noisy, that is, \( \beta \) is close to \( \frac{1}{2} \). Then greater uncertainty about the neighbor herd’s state of health when neither herd is tested constitutes a relatively weak incentive to keep fewer animals since both producers hold similar beliefs about the health of their own and their neighbor’s animals. In contrast, when condition (23) holds and none of the producers are tested, uncertainty about the neighbor herd’s state of health complements uncertainty about the state of health of one’s own animals and incentivizes a (suspicious) \( s \)-producer to keep fewer animals that are more likely to spread the disease.

However, uncertainty about the overall neighbors’ private information decreases as the number of producers \( N \) increases because of our assumption (common in nonspatial epidemiological models) of homogenous mixing in (3). In the limit as \( N \to \infty \), the share of \( s \)-producers and \( h \)-producers converges in probability to \( g_s \) and \( g_h \), respectively, and the impact of the revelation of private information on the laissez-faire depopulation strategies per se vanishes. Nonetheless, as we will show next, the efficient allocation of diagnostic tests and depopulation efforts relies on private information.
6. LARGE NUMBER OF PRODUCERS, $N \to \infty$

Incentives to Report and Targeted Testing under Laissez-Faire Depopulation

Strategic producers take into account how learning about local disease incidence affects the risk of disease transmission from their neighbors (the expected loss from the disease that spreads from a producer’s sick animals to another producer’s initially healthy animals). However, when the number of producers is sufficiently large, the risk of disease transmission is almost beyond the control of any single producer. Then, without transfers, each type of producer always prefers to be selected for testing because it generates the option value of one’s diagnosis and negligibly increases the probability of disease transmission from one’s neighbor. This implies that credible communication necessarily requires transfers in equilibrium with laissez-faire depopulation when the number of producers is large.

Next we will find a transfer scheme that implements the efficient allocation of the diagnostic tests in equilibrium with laissez-faire depopulation as $N \to \infty$ for (i) $\frac{s}{p} < \lambda_s (1 - \delta (1 - \alpha - \frac{M}{N} (1 - \lambda_s)))$ and (ii) $\frac{M}{N} < \min[g_s, g_h]$.\(^{17}\) As in the case of two producers, we begin with equilibrium in period 1. Suppose that it is common knowledge that (a) $M$ s-producers know whether their animals are sick or not, (b) untested $h$- and $s$-producers keep their animals, $n^u(y) = 1, y \in \{s, h\},$ and (c) the tested producers keep healthy animals and remove sick animals: $n^d(\theta) = \theta, \theta \in \{S, H\}$.

Then, applying the law of large numbers, it follows from (3) that the probability that the disease is transmitted to a producer’s healthy animals in period 1 converges to

$$e_s \equiv \Pr(\tilde{\theta}_{i,1} = S \mid \tilde{\theta}_{i,0} = H) \to \delta ((g_s - \frac{M}{N})(1 - \lambda_s) + g_h (1 - \lambda_h)). \quad (24)$$

The first term $(g_s - \frac{M}{N})(1 - \lambda_s)$ in the parentheses on the right-hand side is the share of untested $s$-producers with sick animals, and the second term $g_h (1 - \lambda_h)$ is the share of $h$-producers (all of whom are untested) with sick animals. Now we can easily confirm that the proposed depopulation strategies are, in fact, best responses. An untested producer $i$ earns $\max_{s} p \lambda_{y_i} (1 - e_s) n_i - cn_i$, where $\lambda_{y_i} (1 - e_s)$ is the probability that a producer with signal $y_i$ has healthy animals in period 1. From condition (i) it follows that $n^u(y_i) = 1$ is the best response, $y_i \in \{s, h\}$. Also, producer $i$ who tested negative earns $\max_{n_i} p (1 - e_s) n_i - cn_i$, and $n^d(\theta) = \theta$ is again a privately optimal response, $\theta \in \{S, H\}$.

Now we can calculate the average payoffs in period 1 when producers reveal their private information. When all of the testing capacity is allocated to $s$-producers, and it is commonly known that the share $\frac{M}{N}$ of them learns their test results, as $N \to \infty$, the average payoff converges to

$$\frac{M}{N} \lambda_s ((1 - e_s) p - c) + (g_s - \frac{M}{N})(1 - \lambda_s) p - c) + g_h (1 - e_s) p - c). \quad (25)$$

The first term is the share of the tested initially healthy $s$-producers, $\frac{M}{N} \lambda_s$, times their expected payoff $(1 - e_s) p - c$. The second term is the share of the untested $s$-producers, $g_s - \frac{M}{N}$, times their payoff $\lambda_s (1 - e_s) p - c$. The third term is the share of the untested $h$-producers, $g_h$, times their payoff $\lambda_h (1 - e_s) p - c$. In Appendix B, we find the average payoff when $h$-producers receive the testing priority.

\(^{17}\) The analysis when one or both of these conditions do not hold is similar.
Comparing the average payoffs under the two testing policies confirms that it is socially efficient to test s-producers when condition (i) holds. Furthermore, because (25) is increasing in the testing capacity $M$, testing fewer producers cannot improve welfare. Also, note that (25) can be achieved under any disclosure policy that notifies the tested producers of their test results.

Under the first-best test allocation and disclosure of test results, the incremental payoff (net of transfers) from reporting $s$ rather than $h$ for a producer with signal $y$ converges to

$$\Delta(y) = \frac{M}{g_N} (1 - \lambda_s) c. \quad (26)$$

The first term $\frac{M}{g_N}$ is the probability of being selected for testing, and the second term $(1 - \lambda_s)c$ is the option value generated by one’s diagnosis that consists of the savings $c$ when the animals test positive. To understand (26), note that a producer who reports $h$ will almost surely not be selected for testing and that the reporting decision of a single producer has a vanishingly small impact on the disease transmission probability.

A transfer scheme such that a producer who reports $h$ receives a monetary transfer, $\Delta(h)$, and a producer who reports $s$ receives no transfer makes truthful reporting incentive compatible. Then $h$-producers are indifferent between reporting $h$ and $s$, while $s$-producers strictly prefer to report $s$ because they value diagnosis more than $h$-producers do, $V(s) > V(h)$, when condition (i) holds.

**Efficient Depopulation**

Now, as in Section 4, we consider equilibrium with efficient depopulation and demonstrate that the variability in the costs of compliance can sustain credible reporting without monetary transfers when

$$\lambda_s(1 - \delta(1 - \alpha)(1 - \beta)) \leq \frac{\varepsilon}{p} \leq \frac{\lambda_s - \delta(1 - \alpha)(1 - \beta)(\lambda_s + \lambda_h)}{1 - \lambda_h + \lambda_s}. \quad (27)$$

Note that (27) holds when private signals are sufficiently precise and the disease incursion is sufficiently unlikely. As $\beta \to 1$, (27) converges to $0 \leq \frac{\alpha^2 - \delta(1 - \alpha)^2}{\alpha^2(1 - \alpha)^2}$, which holds for sufficiently large $\alpha$. In Appendix A we establish Lemma 4.

**Lemma 4** Suppose that $N \to \infty$, condition (27) holds, and private information is revealed. Then for sufficiently small $\frac{M}{N}$ in the socially efficient allocation, s-producers receive the testing priority, all untested s-producers depopulate their herds, $n'(s) = 0$, and all h-producers keep theirs, $n'(h) = 1$.

Next we verify that both types of producers prefer to tell the truth when the diagnostic capacity is sufficiently small. The incremental payoff from reporting $s$ rather than $h$ for an s-producer is given by

$$\Delta(s) = \frac{M}{g_N} \lambda_s \underbrace{(1 - \delta(1 - \alpha)(1 - \beta)) p - c}_\text{option value of diagnosis} > 0. \quad (28)$$

An s-producer has nothing to lose from revealing her type, and strictly prefers to tell the truth to increase the odds of being selected for testing. If an s-producer is not selected for testing, the best she can do is to depopulate her herd and earn the reservation payoff of zero (this follows from the first inequality in 27).

The incremental payoff from reporting $s$ rather than $h$ for an h-producer is given by

$$\Delta(h) = \frac{M}{g_N} \lambda_h \underbrace{(1 - \delta(1 - \alpha)(1 - \beta)) p - c}_\text{option value of diagnosis} - \underbrace{(1 - \frac{M}{g_N})(\lambda_h(1 - \delta(1 - \alpha)(1 - \beta)) p - c)}_{\text{incremental cost of compliance}} \leq 0. \quad (29)$$
If an h-producer reports \( s \), she may have to comply with mandatory depopulation if she is not selected for testing, which happens with probability \( 1 - \frac{M}{g_s} \). As the share of the tested producers, \( \frac{M}{N} \), increases, so do the chances of being selected for testing and escaping compliance with mandatory depopulation after reporting \( s \). This tends to make it more difficult to satisfy the incentive compatibility constraint \( \Delta(h) \leq 0 \). Because (29) holds for sufficiently small \( \frac{M}{N} \), we obtain the following:

**Proposition 4** (Credible reporting without transfers with many producers): Suppose that condition (27) holds and \( N \to \infty \). For sufficiently small \( \frac{M}{N} \), the first-best allocation of diagnostic tests is implementable in equilibrium without transfers under mandatory efficient depopulation policy.

However, the variability in the losses from compliance with mandatory depopulation across different types of producers may no longer sustain credible reporting without monetary transfers when the testing capacity is too large. To see why, suppose that the h-producer’s truth-telling incentive compatibility constraint holds with equality, \( \Delta(h) = 0 \), and consider a small increase in the (relative) testing capacity \( \frac{M}{N} \). If the social planner attempts to allocate all of the testing capacity to producers who report \( s \), truthful reporting will no longer be sustainable since constraint \( \Delta(h) \leq 0 \) will not hold. Note that testing some h-producers will restore their incentives to reveal their type. However, such testing policy is not the diagnosticians’s best response after the reports are sent in. Therefore, in the absence of monetary transfers and ability to commit to a particular testing policy, a small increase in the relative testing capacity \( \frac{M}{N} \) can reduce welfare in equilibrium with informative reports. As we will show next, increasing testing capacity unambiguously improves welfare in equilibrium with uninformative reports and a sufficiently large number of producers.

**Random Testing**

Here we show that, unlike in the case with two producers in Section 5 (see Proposition 3), random testing always improves welfare in equilibrium with laissez-faire depopulation and uninformative reports when the number of producers is sufficiently large.\(^{18}\)

**Proposition 5** (Random testing with many producers): Suppose that \( N \to \infty \). In equilibrium with laissez-faire depopulation and uninformative reports, random testing of \( M \) producers is the efficient testing policy for any testing capacity \( M \geq 0 \).

With a sufficiently large number of producers, the positive effects of random testing (more informed depopulation decisions and removal of animals that tested positive) offset the negative effect (suspicious but untested producers keep more animals). The reason is that a small increase in the share of randomly tested producers cannot increase the probability of disease transmission with a sufficient number of producers. As we show in the proof of Proposition 5, there are two possibilities. If each untested producer strictly prefers to either remove or keep her animals, then a small increase in the number of tested producers will decrease the number of untested (and potentially infectious) animals and, therefore, decrease the probability of disease transmission. If some untested producers are indifferent between keeping and removing their animals, then a small increase in the number of tested producers will not change the probability of disease transmission because the indifferent untested producers will increase their herds by just enough to keep the overall level of externality (transmission risk) constant.

---

\(^{18}\) As we pointed out in Section 5, random testing necessarily increases welfare in equilibrium with both uninformative reports and efficient depopulation because the social planner can always ignore the test results. This is true for any \( N \).
7. ASYMMETRIC PRODUCERS

In the previous analysis we considered identical producers. To study how incentives to report and control disease differ among small and large producers, we now assume that producers can own more than one unit of animals (farm). For example, suppose that there are two producers: S (small) and L (large).

Producer S owns 1 unit of animals as in the basic model, and producer L owns the rest \( N - 1 \) units of animals (farms), where \( N \geq 3 \). This has two effects on the efficiency of the disease control decisions. On the one hand, the large producer is more likely to depopulate her herds when it is socially efficient to do so. On the other hand, keeping the total number of animal units (farms) constant, the small producer faces a diminished transmission risk from the farms owned by the large producer and is therefore less likely to depopulate her herd in a socially efficient manner.

Turning to the reporting incentives, the small producer tends to take the overall transmission risk as given since the average disease incidence on the farms owned by the large producer is more stable than on her own farm. Therefore, the small producer prefers to be selected for testing as long as the cost of compliance with the mandatory disease control program, if any, is not too onerous. The large producer may also prefer to educate the small producer about the initial state of health of her animals (that is, to test animals on the small farm) so as to decrease the probability of becoming infected by the small producer (who exercises, on average, less care than the large producer). On the other hand, the large producer may value information about the initial state of health of her animals more than the small producer because producer L can coordinate disease control of multiple units more efficiently than \( N - 1 \) noncooperative small producers. Therefore, the incremental payoff from being selected for testing for the large producer may be greater or smaller than that for the small producer.

A novel feature of the model with asymmetric producers is the variable report credibility whereas some but not all producers send informative reports in equilibrium. For sufficiently large \( N \), in equilibrium without transfers it is efficient to rely on the reports submitted by the large producer and ignore the report submitted by the small producer. The interests of the large producer are almost perfectly aligned with that of the social planner and the large producer is well incentivized to reveal her observations of local health conditions. However, the small producer (rationally) ignores the social benefits of truthfully revealing her private information.
8. CONCLUSIONS

Our model of reporting suspicions and early detection of a controllable infectious disease focuses on the informational consequences of reporting. The previous literature views reports of local disease incidence as signals that determine the cost of compliance with an exogenous government disease control policy. In our model reports do not directly affect payoffs but are inputs in the surveillance program that may or may not include mandatory disease control. We have found that even without mandatory depopulation, the incentives to manipulate a disease surveillance program exist and better information about local disease incidence is valued differently by different producers.

This paper has shown how the implementation of an efficient allocation of a diagnostic resource based on cheap talk with multiple senders depends on the details of the environment such as the number of senders, the likelihood of disease occurrence, and the precision of private information as well as institutional characteristics such as the feasibility of mandatory disease control, diagnostic capacity, and the cost of administering monetary transfers. With a small number of well-informed senders and unlikely occurrence of disease, there exists an equilibrium in which reports are credible without transfers and mandatory disease control. In this situation, disease-suspecting senders have a testing priority. From a societal point of view, a nonsuspecting sender is unlikely to be infectious, but a suspecting sender imposes a greater externality and is more likely to benefit from diagnosis. From a sender’s point of view, a suspecting sender is more concerned with the health of her own animals than becoming infected by her neighbor’s animals and prefers to be diagnosed, but the opposite is true for a nonsuspecting sender. Then truth telling is incentive compatible for each type of sender because the private and social preferences over the allocation of the diagnostic tests, on average, agree.

However, the senders’ and social planner’s preferences over the allocation of the diagnostic tests diverge when there are many senders, when disease occurrence is likely, or when private information is noisy. For example, when the number of senders is large, an individual sender is more concerned with state of health of her own animals because she has little control over the probability that the disease will spread. Then monetary transfers are needed to incentivize the revelation of private information that reduces one’s chances of learning more about local disease prevalence. We have shown that although private information revelation is not necessary for testing to improve welfare when the number of producers is large, testing without credible communication of suspicions can decrease welfare under laissez-faire disease control when the number of producers is small and their signals are sufficiently precise.

In reality, the news of a confirmed case of a notifiable infectious disease is typically followed by enhanced surveillance and disease control programs. We have shown that mandatory efficient depopulation may make it easier or more difficult to sustain credible reporting or may obviate the need to rely on reports altogether. Nonetheless, costly compliance with the mandatory efficient depopulation policy sustains credible communication and first-best targeted testing without transfers when private information is precise and the disease occurrence is unlikely, and the diagnostic capacity is sufficiently small relative to the number of producers. The model generates an empirically testable hypothesis that less underreporting occurs when farmers are more knowledgeable about the disease and the disease incursion is less likely.

In our model laissez-faire depopulation strategies of tested farmers are socially efficient, and the negative externalities arise solely due to uncertainty about initial disease incidence. A more general model would take into account a variety of socially inefficient actions that farmers can take after they find out that their animals are sick (Olmstead and Rhode 2004). In our setting this can be done by assuming that farmers can dispose of their diseased or suspected animals by selling them rather than presenting them for culling and complying with biosecurity measures such as sanitizing and disinfecting facilities (World Bank et al. 2006). Our model can also be used to study the effects of early detection on the preventive actions by allowing producers to control the probability that their animals are healthy in period 0, \( \alpha \), but
at a cost. Another interesting extension is to consider more general private information structures and reports that are partially informative.

The assumption that the disease does not spread between farms (senders) in period 0 can be relaxed by allowing the initial states of health, \( \theta_{i,0} \), to be positively correlated across farms. Then learning about the initial health of animals on tested farms will be directly informative about the initial health of animals on untested farms. This tends to decrease the option value generated by the diagnostic tests for the farmers whose animals are tested because the tests on neighboring farms provide some information to the untested farmers about their own animals. Also, the variability in the costs of compliance with mandatory depopulation for tested and untested farmers tends to diminish when the initial disease statuses of animals on different farms are correlated. While the smaller option value generated by the individual diagnosis reduces, less variability in the compliance costs between tested and untested farmers increases the incentives to send a report that raises one’s probability of being selected for testing. Therefore, correlation among \( \theta_{i,0} \)'s can make it easier or more difficult to sustain credible reporting. An interesting topic for future research is to consider the design of a compensation scheme that incentivizes truthful reporting of clinically suspect cases and the allocation of diagnostic efforts in a fully dynamic susceptible–infected–recovered model (SIR) of an infectious disease (Anderson and May 1979).
APPENDIX: PROOFS

Proof of Lemma 2: First, we establish an upper bound on the expected payoffs when none of the producers are tested. Second, we show that the producers can achieve greater payoffs when one of them is tested.

Step 1. If none the producers are tested, their expected payoffs cannot exceed those achieved by the social planner who knows the realizations of the private signals \((y_1, y_2)\) and controls the number of animals kept by each producer \((n_1, n_2)\):

\[
W^s(y_1, y_2, \emptyset) = \max_{n_1, n_2 \in [0,1]} p_1 n_1 \lambda_{y_1} (1 - \delta n_2 (1 - \lambda_{y_2})) - cn_1 + p_2 n_2 \lambda_{y_2} (1 - \delta n_1 (1 - \lambda_{y_1})) - cn_2
\]

Therefore, we have

\[
W(y_1, y_2, \emptyset) \leq W^s(y_1, y_2, \emptyset) = \max[0, p \max[\lambda_{y_1}, \lambda_{y_2}]] - c,
\]

\[
p(\lambda_{y_1} (1 - \delta(1 - \lambda_{y_2}))) - c + p\lambda_{y_2} (1 - \delta(1 - \lambda_{y_1})) - c\].
\] (A1)

Step 2. Note that it must be that

\[
\max[W(y_1, y_2, 1), W(y_1, y_2, 2)] \geq W^s(y_1, y_2, \emptyset)
\] (A2)

when \(\tilde{\pi}_p \in [\lambda_{y} - \delta(1 - \lambda_{y}), \lambda_{y}]\) and \(\tilde{\pi}_p \in [\lambda_{y} - \delta(1 - \lambda_{y}), \lambda_{y}]\) because noncooperative producers choose the socially efficient herd sizes, and social welfare cannot decrease when the social planner learns the initial state of health of the animals of one of the producers. It is also easy to verify that (A2) holds for \(\tilde{\pi}_p \in (\lambda_{y} - \delta(1 - \lambda_{y}), \lambda_{y})\) or \(\tilde{\pi}_p \in (\lambda_{y} - \delta(1 - \lambda_{y}), \lambda_{y})\). There are two cases to consider. If it is socially efficient to let both producers keep their animals when neither producer is tested, then it is clearly socially efficient to let one of the producers know her animals’ health state since this will reduce the transmission risk and allow the tested producer to make an informed depopulation decision. So suppose that

\[
W^s(y_1, y_2, \emptyset) = p \max[\lambda_{y_1}, \lambda_{y_2}] - c \geq 0
\]. Then (A2) holds because

\[
\max[W(y_1, y_2, 1), W(y_1, y_2, 2)] \geq \max[(1 + \lambda_{y_1})(\lambda_{y_2} p - c), (1 + \lambda_{y_2})(\lambda_{y_1} p - c)]
\]

\[
> p \max[\lambda_{y_1}, \lambda_{y_2}] - c = W^s(y_1, y_2, \emptyset).
\]

Therefore, by Lemma 1, it must be that the producers can achieve higher payoffs when one of them is tested than when neither is tested for any beliefs that producers may have about one another’s private information. \(Q.E.D.\)

Proof of Lemma 4: When the share \(\frac{M}{N} > 0\) of \(s\)-producers are tested the average payoff,

\[
w^s(n^s(s), n^s(h), s),\]

where the third argument is the type of producers that are targeted for testing, is given by

\[
w^s(n^s(s), n^s(h), s) = \frac{M}{N} \lambda_{s} ((1 - e_s) p - c) + n^s(s)(g_s - \frac{M}{N})(\lambda_{s} (1 - e_s) p - c)
\]

\[
+ n^s(h)g_h(\lambda_{h} (1 - e_s) p - c),
\]

where \(e_s = \delta(n^s(s)(g_s - \frac{M}{N})(1 - \lambda_{s}) + n^s(h)g_h (1 - \lambda_{h})).\)
The first-order conditions for the socially optimal number of animals kept by the two types of untested producers evaluated at \( n^s(s) = 0 \) and \( n^s(h) = 1 \) are given by (A3a)

\[
\frac{\partial w^s(0,1,s)}{\partial n^s(s)} = (g_s - \frac{M}{N})(\lambda_s (1 - \delta g_s (1 - \lambda_s)) p - c) - (\frac{M}{N} \lambda_s + g_s \lambda_s) p \delta(g_s - \frac{M}{N} (1 - \lambda_s)) \leq 0, \tag{A3a}
\]

\[
\frac{\partial w^s(0,1,h)}{\partial n^s(h)} = g_h (\lambda_h (1 - \delta g_h (1 - \lambda_h)) p - c) - (\frac{M}{N} \lambda_h + g_h \lambda_h) p \delta g_h (1 - \lambda_h) \geq 0 \tag{A3b}
\]

holds by the first inequality in (27). For \( \frac{M}{N} = 0 \), inequality in (A3b) can be rewritten as

\[ \frac{\lambda}{p} \leq \lambda_h (1 - 2 \delta g_h (1 - \lambda_s)) \],

which is implied by the second inequality in (27).

Similarly, it is easy to check that when the share \( \frac{M}{N} \) of \( h \)-producers is tested, welfare is also maximized at \( n^s(s) = 0 \) and \( n^s(h) = 1 \) as long as condition (27) holds. So the maximum average payoff when \( h \)-producers receive the testing priority, \( w^s(0,1,h) \), is given by

\[
w^s(0,1,h) = \frac{M}{N} \lambda_h ((1 - e_h) p - c) + (g_h - \frac{M}{N} \lambda_h (1 - e_h) p - c),
\]

where \( e_h = \delta (g_h - \frac{M}{N}) (1 - \lambda_h) \). From the second inequality in (27) it follows that \( w^s(0,1,s) > w^s(0,1,h) \) when \( \frac{M}{N} \) is sufficiently close to 0.

**Q.E.D.**

**Proof of Proposition 5:** Following the same steps as in Section 5, it is easy to verify that in equilibrium with uninformative reports and random testing, the depopulation strategy that constitutes a best response for an untested \( h \)-producer is given by

\[
n^u(h) = \begin{cases} 0, & \text{if } \frac{\lambda}{p} \geq \lambda_h \\ \tilde{n}_h, & \text{if } \lambda_h (1 - \delta (1 - \frac{M}{N})(1 - \alpha)(1 - \beta)) \leq \frac{\lambda}{p} < \lambda_h \\ 1, & \text{if } \frac{\lambda}{p} < \lambda_h (1 - \delta (1 - \frac{M}{N})(1 - \alpha)(1 - \beta)) \end{cases},
\]

where \( \tilde{n}_h \in (0,1] \) solves the indifference condition \( \lambda_h (1 - \delta (1 - \frac{M}{N}) g_h (1 - \lambda_h) \tilde{n}_h) = \frac{\lambda}{p} \), and a best-response depopulation for an \( s \)-producer is given by

\[
n^u(s) = \begin{cases} 0, & \text{if } \frac{\lambda}{p} \geq \lambda_s (1 - \delta (1 - \frac{M}{N})(1 - \alpha)(1 - \beta)) \\ \tilde{n}_s, & \text{if } \lambda_s (1 - \delta (1 - \frac{M}{N})(1 - \alpha)) \leq \frac{\lambda}{p} < \lambda_s (1 - \delta (1 - \frac{M}{N})(1 - \alpha)(1 - \beta)) \\ 1, & \text{if } \frac{\lambda}{p} < \lambda_s (1 - \delta (1 - \frac{M}{N})(1 - \alpha)) \end{cases},
\]

where \( \tilde{n}_s \in (0,1] \) solves the indifference condition \( \lambda_s (1 - \delta (1 - \frac{M}{N}) (g_s (1 - \lambda_s) + g_s (1 - \lambda_s) \tilde{n}_s) = \frac{\lambda}{p} \).

Also, in equilibrium a producer who tested negative keeps her animals, and a producer who tested positive removes her animals. Therefore, as \( N \to \infty \), the average payoff \( \lim_{N \to \infty} \frac{E^{\mathbb{W}(N)}(Y_1, ..., Y_N, 1,..., M^1)}{N} \), under random testing of \( M^1 \) producers converges to
\[
\sum_{y \in \{r,s\}} g_y \lambda_y ((1-e_r) p - c) + (1 - \frac{M}{N}) g_s (\lambda_s (1-e_s) p - c)
\]

where \( e_r = \delta \sum_{y \in \{r,s\}} g_y (1 - \frac{M}{N}) (1 - \lambda_y) n^y \) is the probability of becoming infected in period 1.

Substituting (A4) and (A5) in (A6), the average payoff (welfare) becomes

\[
\begin{cases}
\alpha (1 - \delta \alpha) p - c + \bar{m} \alpha (c \bar{p} + c), & \text{if } \frac{\bar{p}}{\bar{m}} < \lambda_y (1 - \delta (1 - \bar{m}) \alpha) \\
(c(\bar{m}(1 - \alpha^2) + \frac{\alpha^2}{M} - \alpha \beta - \bar{m} \beta)), & \text{if } \lambda_y (1 - \delta (1 - \bar{m}) \alpha) \leq \frac{\bar{p}}{\bar{m}} < \lambda_y (1 - \delta (1 - \bar{m}) \alpha \beta) \\
\alpha (\bar{m} \beta + \beta) (1 - \delta (1 - \bar{m}) \alpha \beta) p - c(\bar{m}(2 \alpha - 1) + \alpha \beta + \bar{m} \beta), & \text{if } \lambda_y (1 - \delta (1 - \bar{m}) \alpha \beta) \leq \frac{\bar{p}}{\bar{m}} < \lambda_y (1 - \delta (1 - \bar{m}) \alpha \beta) \\
\bar{m} c \alpha (\frac{1}{\lambda_y} - 1), & \text{if } \lambda_y (1 - \delta (1 - \bar{m}) \alpha \beta) \leq \frac{\bar{p}}{\bar{m}} < \lambda_y \\
\end{cases}
\]

where \( \alpha = 1 - \alpha \), \( \beta = 1 - \beta \), and \( \bar{m} = \frac{M}{N} \). Because the average payoff is monotonically increasing in \( \bar{m} \), it follows that random testing increases welfare when the number of producers is sufficiently large and reports are uninformative.

**Equilibrium Average Payoff When h-Producers Receive a Testing Priority**

Now suppose that h-producers receive a testing priority. There are two cases to consider. For \( \frac{\bar{p}}{\bar{m}} \leq \lambda_y (1 - \delta (1 - \alpha - \frac{M}{N} (1 - \lambda_y))) \) both types of untested producers keep their animals. Then the disease transmission probability converges to

\[
e_h \equiv \delta (g_h - \frac{M}{N})(1 - \lambda_h) + g_s (1 - \lambda_s)
\]

and the average payoff converges to

\[
\frac{M}{N} \lambda_h ((1-e_h) p - c) + (g_h - \frac{M}{N}) (\lambda_h (1-e_h) p - c) + g_s (\lambda_s (1-e_s) p - c)
\]

For \( \frac{\bar{p}}{\bar{m}} > \lambda_y (1 - \delta (1 - \alpha - \frac{M}{N} (1 - \lambda_y))) \) s-producers keep, on average, \( n_s \in (0,1) \) animals. Then the transmission probability becomes

\[
e_h = \delta (g_h - \frac{M}{N})(1 - \lambda_h) + n_s g_s (1 - \lambda_s)
\]

s-producers are indifferent about keeping or removing their animals if \( n_s \) solves the indifference condition \( p \lambda_s (1 - \delta ((1-\alpha)(1-\beta + n_s, \beta) - \frac{M}{N} (1 - \lambda_h))) - c = 0 \).

The difference between (B1) and (B3) is that there are fewer sick animals kept by untested s-producers. The average payoff now becomes

\[
\frac{M}{N} \lambda_h ((1-e_h) p - c) + (g_h - \frac{M}{N}) (\lambda_h (1-e_h) p - c) + n_s g_s (\lambda_s (1-e_s) p - c)
= (\frac{M}{N} (1 - \lambda_h) + g_h (\frac{\lambda_h}{\lambda_s} - 1)) c
\]

where we substituted for \( n_s \) to obtain the last equality.
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